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A NOTE ON q-ANALOGUE OF CATALAN NUMBERS ASSOCIATED WITH q-CHANGHEE NUMBERS

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Abstract: In this paper, we study q-analogue of Catalan numbers and polynomials by using p-adic q-integral on $\mathbb{Z}_p$. We investigate some properties of these numbers and polynomials. In addition, we define q-analogue of $\frac{1}{2}$-Changhee numbers by using p-adic q-integral on $\mathbb{Z}_p$ and derive their explicit expressions and some identities involving them.

Keywords and Phrases: Catalan numbers, $\frac{1}{2}$-Changhee numbers, q-Catalan numbers, q-analogue of $\frac{1}{2}$-Changhee numbers, q-Euler numbers.

2020 Mathematics Subject Classification: 11B68, 11S40, 11S80.

1. Introduction

Let p be a fixed odd prime number. Throughout this paper, $\mathbb{Z}_p$, $\mathbb{Q}_p$ and $\mathbb{C}_p$ will denote the ring of p-adic integers, the filed of p-adic rational numbers and the completion of an algebraic closure of $\mathbb{Q}_p$. The p-adic norm $|\ .\ |_p$ is normalized by $|\ p\ |_p = \frac{1}{p}$. Let $C(\mathbb{Z}_p)$ be the space of continuous function on $\mathbb{Z}_p$. Let q be an indeterminate in $\mathbb{C}_p$ with $|\ 1-q\ |_p < 1$ and q-extension of x is defined by $[x]_q = \frac{1-q^x}{1-q}$.

Then the fermionic p-adic q-integral of f on $\mathbb{Z}_p$ is defined by Kim as follows

$$I_{-q}(f) = \int_{\mathbb{Z}_p} f(x)d\mu_{-q}(x) = \lim_{N\to\infty} \sum_{x=0}^{p^N-1} f(x)\mu_{-q}(x+p^N\mathbb{Z}_p),$$

$$= \lim_{N\to\infty} \frac{1}{[p^N]_{-q}} \sum_{x=0}^{p^N-1} f(x)(-q)^x, \quad \text{(see [4, 10, 14, 15, 26]).} \tag{1.1}$$

Let $f_1(x) = f(x+1)$. Then, by (1.1), we get

$$qI_{-q}(f_1) + I_{-q}(f) = [2]_q f(0). \tag{1.2}$$

It is well known that the Euler numbers are defined by

$$\frac{2}{e^t+1} = \sum_{n=0}^{\infty} E_n \frac{t^n}{n!}, \quad \text{(see [12, 17, 19]).} \tag{1.3}$$

Let q be an indeterminate in $\mathbb{C}_p$ with $\mid 1-q \mid_p < 1$. The q-analogues of Euler numbers are given by

$$\frac{[2]_q}{qe^t+1} = \sum_{n=0}^{\infty} E_{n,q} \frac{t^n}{n!}, \quad \text{(see [7, 23]).} \tag{1.4}$$

Note that $\lim_{q\to 1} E_{n,q} = E_n, (n \geq 0)$.
The q-analogues of Changhee numbers are given by

$$\frac{[2]_q}{[2]_q+t} = \sum_{n=0}^{\infty} Ch_{n,q} \frac{t^n}{n!}, \quad \text{(see [6-10, 13, 14]).} \tag{1.5}$$

Kim *et al.* [8] introduced the λ-Changhee polynomials defined by

$$\frac{2}{(1+t)^{\lambda}+1}(1+t)^{\lambda x} = \sum_{n=0}^{\infty} Ch_{n,\lambda}(x) \frac{t^n}{n!}, \tag{1.6}$$

where $\lambda \in \mathbb{Z}_p$.
When $x = 0$, $Ch_{n,\lambda} = Ch_{n,\lambda}(0)$ are called the λ-Changhee numbers.
For $n \geq 0$, the Stirling numbers of the first kind are defined by

$$(x)_n = \sum_{l=0}^{n} S_1(n,l)x^l, \quad \text{(see [1-15])} \tag{1.7}$$

where $(x)_0 = 1$, and $(x)_n = x(x-1)\cdots(x-n+1), (n \geq 1)$. From (1.7), it is easy to see that

$$\frac{1}{r!}(\log(1+t))^r = \sum_{n=r}^{\infty} S_1(n,r)\frac{t^n}{n!}, \quad (r \geq 0), \quad \text{(see [11-20])}. \tag{1.8}$$

For $n \geq 0$, the Stirling numbers of the second kind are defined by

$$x^n = \sum_{l=0}^{n} S_2(n,l)(x)_l, \quad \text{(see [15-27])}. \tag{1.9}$$

From (1.9), we see that

$$\frac{1}{r!}(e^t - 1)^r = \sum_{n=r}^{\infty} S_2(n,r)\frac{t^n}{n!}. \tag{1.10}$$

As is well known, the Catalan numbers are defined by the generating function as follows (see [1, 2, 3, 20, 21, 22, 24, 25, 27])

$$\frac{2}{1+\sqrt{1-4t}} = \frac{1-\sqrt{1-4t}}{2t} = \sum_{n=0}^{\infty} C_n t^n, \tag{1.11}$$

where $C_n = \binom{2n}{n}\frac{1}{n+1}, (n \geq 0)$.
The Catalan polynomials are defined by the generating function as follows (see [13])

$$\begin{aligned}\int_{\mathbb{Z}_p} (1-4t)^{\frac{x+y}{2}} d\mu_{-1}(y) &= \frac{2}{1+\sqrt{1-4t}}(1-4t)^{\frac{x}{2}} \\ &= \sum_{n=0}^{\infty} C_n(x)t^n. \end{aligned} \tag{1.12}$$

When $x = 0$, $C_n = C_n(0)$ are called the Catalan numbers.
Thus, by (1.11) and (1.12), we have

$$C_n(x) = \sum_{m=0}^{n}\sum_{j=0}^{m} \left(\frac{x}{2}\right)^j S_1(m,j)(-4)^m \frac{C_{n-m}}{m!}.$$

Kim introduced the $\frac{1}{2}$-Changhee polynomials which are given by the generating function (see [12])

$$\int_{\mathbb{Z}_p} (1+t)^{\frac{x+y}{2}} d\mu_{-1}(y) = \frac{2}{1+\sqrt{1+t}}\sqrt{(1+t)^x}$$

$$= \sum_{n=0}^{\infty} Ch_{n,\frac{1}{2}}(x)\frac{t^n}{n!}. \tag{1.13}$$

When $x = 0$, $Ch_{n,\frac{1}{2}} = Ch_{n,\frac{1}{2}}(0)$ are called the $\frac{1}{2}$-Changhee numbers.
On replacing t by $-4t$ in (1.13) and by using (1.12), we have

$$\int_{\mathbb{Z}_p} (1-4t)^{\frac{x+y}{2}} d\mu_{-1}(y) = \frac{2}{1+\sqrt{1-4t}}\sqrt{(1-4t)^x}$$

$$= \sum_{n=0}^{\infty} Ch_{n,\frac{1}{2}}(x)(-4)^n\frac{t^n}{n!}.$$

$$\sum_{n=0}^{\infty} C_n(x)t^n = \sum_{n=0}^{\infty} Ch_{n,\frac{1}{2}}(x)(-4)^n\frac{t^n}{n!}. \tag{1.14}$$

Comparing the coefficients of t, we get

$$C_n(x) = \frac{(-1)^n}{n!} Ch_{n,\frac{1}{2}}(x)2^{2n}.$$

Recently, Kim *et al.* [11] introduced the q-analogues of Catalan polynomials which are given by

$$\int_{\mathbb{Z}_p} (1-4t)^{\frac{x+y}{2}} d\mu_{-q}(y) = \frac{[2]_q}{1+q\sqrt{1-4t}}(1-4t)^{\frac{x}{2}}$$

$$= \sum_{n=0}^{\infty} C_{n,q}(x)t^n. \tag{1.15}$$

When $x = 0$, $C_{n,q} = C_{n,q}(0)$ are called the q-Catalan numbers.

The aim of the paper is to introduce the q-analogues of Catalan numbers $C_{n,q}$ with the help of a p-adic q-integral on $\mathbb{Z}_p$ and derive explicit expressions and some identities for those numbers. In more detail, we deduce explicit expressions of $C_{n,q}$, as a rational function in terms of Euler number and Stirling numbers of the first kind, as a fermionic p-adic q-integral on $\mathbb{Z}_p$ and involving q-analogue of $\frac{1}{2}$-Changhee numbers.

2. q-analogue Catalan Numbers Associated with q-Changhee Numbers

In this section, we assume that $q, t \in \mathbb{C}_p$ with $|1-q| < 1$ and $|t| < p^{-\frac{1}{p-1}}$. Let us apply (1.2) with $f(x) = (1+t)^{\frac{1}{2}}$. Then, we have

$$\int_{\mathbb{Z}_p} (1+t)^{\frac{x}{2}} d\mu_{-q}(x) = \frac{[2]_q}{1+q\sqrt{1+t}} = \frac{[2]_q}{1-q^2-q^2t}(1-q\sqrt{1+t}). \tag{2.1}$$

Now, we consider the q-analogues of $\frac{1}{2}$-Changhee numbers which are defined by

$$\frac{[2]_q}{1-q^2-q^2t}(1-q\sqrt{1+t})=\sum_{n=0}^{\infty}Ch_{n,q,\frac{1}{2}}\frac{t^n}{n!}. \tag{2.2}$$

Note that

$$\lim_{q\to 1}Ch_{n,q,\frac{1}{2}}=Ch_{n,\frac{1}{2}}\quad (n\geq 0).$$

From (1.2), we note that

$$\int_{\mathbb{Z}_p}e^{xt}d\mu_{-q}(x)=\frac{[2]_q}{qe^t+1}=\sum_{n=0}^{\infty}E_{n,q}\frac{t^n}{n!}. \tag{2.3}$$

Thus, by (2.3), we get

$$\int_{\mathbb{Z}_p}x^n d\mu_{-q}(x)=E_{n,q},(n\geq 0). \tag{2.4}$$

On the other hand, by using (2.4) we also have

$$\begin{aligned}\int_{\mathbb{Z}_p}(1+t)^{\frac{x}{2}}d\mu_{-q}(x)&=\sum_{m=0}^{\infty}\int_{\mathbb{Z}_p}x^m d\mu_{-q}(x)\frac{1}{m!}(\log(1+t))^m\\&=\sum_{m=0}^{\infty}E_{m,q}2^{-m}\sum_{n=m}^{\infty}S_1(n,m)\frac{t^n}{n!}\\&=\sum_{n=0}^{\infty}\left(\sum_{m=0}^{n}E_{m,q}2^{-m}S_1(n,m)\right)\frac{t^n}{n!}.\end{aligned} \tag{2.5}$$

Therefore, by (2.2) and (2.5), we obtain the following theorem.

Theorem 2.1. *For $n\geq 0$, we have*

$$Ch_{n,q,\frac{1}{2}}=\sum_{m=0}^{n}E_{m,q}2^{-m}S_1(n,m).$$

By replacing t by $-4t$ in (2.2), we have

$$\int_{\mathbb{Z}_p}(1-4t)^{\frac{x}{2}}d\mu_{-q}(x)=\frac{[2]_q}{q\sqrt{1-4t}+1}=\sum_{n=0}^{\infty}C_{n,q}t^n. \tag{2.6}$$

On the other hand,

$$\int_{\mathbb{Z}_p}(1-4t)^{\frac{x}{2}}\,d\mu_{-q}(x)=\sum_{n=0}^{\infty}Ch_{n,q,\frac{1}{2}}\frac{(-4)^n t^n}{n!}. \tag{2.7}$$

Therefore, by (2.6) and (2.7), we obtain the following theorem.

Theorem 2.2. *For $n \geq 0$, we have*

$$C_{n,q}=(-1)^n\frac{4^n Ch_{n,q,\frac{1}{2}}}{n!}.$$

From (2.1), we observe that

$$\begin{aligned}\int_{\mathbb{Z}_p}(1+t)^{\frac{x}{2}}\,d\mu_{-q}(x)&=\sum_{m=0}^{\infty}\int_{\mathbb{Z}_p}\binom{\frac{x}{2}}{m}d\mu_{-q}(x)\frac{[\log(1+t)]^m}{m!}\\&=\sum_{m=0}^{\infty}C_{m,q}\sum_{n=m}^{\infty}S_1(n,m)\frac{t^n}{n!}\\&=\sum_{n=0}^{\infty}\left(\sum_{m=0}^{n}C_{m,q}S_1(n,m)\right)\frac{t^n}{n!}.\end{aligned} \tag{2.8}$$

Therefore, by (2.2) and (2.8), we get the following theorem.

Theorem 2.3. *For $n \geq 0$, we have*

$$Ch_{n,q,\frac{1}{2}}=\sum_{m=0}^{n}C_{m,q}S_1(n,m).$$

First, we note that

$$\begin{aligned}(1+t)^{\frac{1}{2}}&=\sum_{n=0}^{\infty}\binom{\frac{1}{2}}{n}t^n=\sum_{n=0}^{\infty}\frac{(\frac{1}{2})_n}{n!}t^n\\&=\sum_{n=0}^{\infty}\frac{1(\frac{1}{2}-1)(\frac{1}{2}-2)\cdots(\frac{1}{2}-n+1)}{n!}t^n\\&=\sum_{n=0}^{\infty}\frac{(-1)^{n-1}1.3.5\cdots(2n-3)}{n!2^n}t^n\\&=\sum_{n=0}^{\infty}\frac{(-1)^{n-1}1.3.4\cdots(2n-3)(2n-2)(2n-1)(2n)}{n!2^n 2.4.6\cdots(2n-2)(2n-1)(2n)}t^n\end{aligned}$$

$$= \sum_{n=0}^{\infty}(-1)^{n-1}(-1)^{n-1}\frac{(2n)!}{n!4^n(2n-1)n!}t^n = \sum_{n=0}^{\infty}\binom{2n}{n}\frac{(-1)^{n-1}}{4^n(2n-1)}t^n. \tag{2.9}$$

By (2.1) *and* (2.9), *we get*

$$[2]_q = \left(\sum_{n=0}^{\infty} Ch_{n,q,\frac{1}{2}}\frac{t^n}{n!}\right)\left((1+q\sqrt{1+t})\right)$$

$$= \sum_{n=0}^{\infty} Ch_{n,q,\frac{1}{2}}\frac{t^n}{n!} + \left(\sum_{n=0}^{\infty} Ch_{n,q,\frac{1}{2}}\frac{t^n}{n!}\right)\left(q\sum_{m=0}^{\infty}\binom{2m}{m}\frac{(-1)^{m-1}}{4^m(2m-1)}t^m\right)$$

$$= \sum_{n=0}^{\infty} Ch_{n,q,\frac{1}{2}}\frac{t^n}{n!} + \sum_{n=0}^{\infty}\left(q\sum_{m=0}^{n} C_m\frac{(m+1)(-1)^{m-1}}{4^m(2m-1)}\frac{m!n!}{(n-m)!m!}Ch_{n-m,q,\frac{1}{2}}\right)\frac{t^n}{n!}$$

$$= \sum_{n=0}^{\infty} Ch_{n,q,\frac{1}{2}}\frac{t^n}{n!} + \sum_{n=0}^{\infty}\left(q\sum_{m=0}^{n} C_m\frac{(m+1)(-1)^{m-1}}{4^m(2m-1)}\binom{n}{m}Ch_{n-m,q,\frac{1}{2}}\right)\frac{t^n}{n!}. \tag{2.10}$$

By comparing the coefficients of t *on both sides, we obtain the following theorem.*

Theorem 2.4. *For* $n \geq 0$, *we have*

$$Ch_{n,q,\frac{1}{2}} + q\sum_{m=0}^{n} C_m\frac{(m+1)(-1)^{m-1}}{4^m(2m-1)}\binom{n}{m}Ch_{n-m,q,\frac{1}{2}} = \begin{cases} [2]_q, & if n = 0 \\ 0, & if n > 1. \end{cases}$$

By replacing t *by* $-\frac{t}{4}$ *in* (1.4), *we get*

$$\frac{[2]_q}{1+q\sqrt{1+t}} = \sum_{n=0}^{\infty} C_{n,q}\left(-\frac{t}{4}\right)^n. \tag{2.11}$$

$$= \sum_{n=0}^{\infty} C_{n,q}(-1)^n 4^{-n} t^n$$

On the other hand, we have

$$\frac{[2]_q}{1+q\sqrt{1+t}} = \sum_{n=0}^{\infty} Ch_{n,q,\frac{1}{2}}\frac{t^n}{n!}. \tag{2.12}$$

Therefore, by (2.11) *and* (2.12), *we get the following theorem.*

Theorem 2.5. *For $n \geq 0$, we have*

$$Ch_{n,q,\frac{1}{2}} = n!C_{n,q}(-1)^n 2^{-2n}.$$

Replacing t by $e^{2t}-1$ in (2.1), we have

$$\begin{aligned}
\int_{\mathbb{Z}_p} e^{xt} d\mu_{-q}(x) &= \frac{[2]_q}{qe^t+1} = \sum_{m=0}^{\infty} Ch_{m,q,\frac{1}{2}} \frac{(e^{2t}-1)^m}{m!} \\
&= \sum_{m=0}^{\infty} Ch_{m,q,\frac{1}{2}} \sum_{n=m}^{\infty} S_2(n,m) \frac{2^n t^n}{n!} \\
&= \sum_{n=0}^{\infty} \left(\sum_{m=0}^{n} 2^n Ch_{m,q,\frac{1}{2}} S_2(n,m) \right) \frac{t^n}{n!}. \qquad (2.13)
\end{aligned}$$

On the other hand, we have

$$\int_{\mathbb{Z}_p} e^{xt} d\mu_{-q}(x) = \sum_{n=0}^{\infty} E_{n,q} \frac{t^n}{n!}. \qquad (2.14)$$

Therefore, by (2.13) and (2.14), we obtain the following theorem.

Theorem 2.6. *For $n \geq 0$, we have*

$$E_{n,q} = \sum_{m=0}^{n} 2^n Ch_{m,q,\frac{1}{2}} S_2(n,m).$$

Now, we observe that

$$\begin{aligned}
(1+t)^{\frac{x}{2}} &= \sum_{m=0}^{\infty} \left(\frac{x}{2}\right)^m \frac{[\log(1+t)]^m}{m!} \\
&= \sum_{m=0}^{\infty} \left(\frac{x}{2}\right)^m \sum_{n=m}^{\infty} S_1(n,m) \frac{t^n}{n!} \\
&= \sum_{n=0}^{\infty} \left(\sum_{m=0}^{n} \left(\frac{x}{2}\right)^m S_1(n,m) \right) \frac{t^n}{n!}. \qquad (2.15)
\end{aligned}$$

Now, we consider the q-analogues of $\frac{1}{2}$-Changhee polynomials which are given by the generating function to be

$$\int_{\mathbb{Z}_p} (1+t)^{\frac{x+y}{2}} d\mu_{-q}(y) = \frac{[2]_q}{1+q\sqrt{1+t}} \sqrt{(1+t)^x}$$

$$= \sum_{n=0}^{\infty} Ch_{n,q,\frac{1}{2}}(x)\frac{t^n}{n!}. \tag{2.16}$$

When $x = 0$, $Ch_{n,q,\frac{1}{2}} = Ch_{n,q,\frac{1}{2}}(0)$ *are called the q-analogues of* $\frac{1}{2}$*-Changhee numbers.*

From (2.16), *we note that*

$$\sum_{n=0}^{\infty} Ch_{n,q,\frac{1}{2}}(x)\frac{t^n}{n!} = \frac{[2]_q}{1+q\sqrt{1+t}}\sqrt{(1+t)^x}$$

$$= \left(\sum_{n=0}^{\infty} Ch_{n,q,\frac{1}{2}}\frac{t^n}{n!}\right)\left(\sum_{l=0}^{\infty}\sum_{m=0}^{l}\left(\frac{x}{2}\right)^m S_1(l,m)\frac{t^l}{l!}\right)$$

$$= \sum_{n=0}^{\infty}\left(\sum_{m=0}^{n}\binom{n}{l}Ch_{n-l,q,\frac{1}{2}}\sum_{m=0}^{l}\left(\frac{x}{2}\right)^m S_1(l,m)\right)\frac{t^n}{n!}. \tag{2.17}$$

By (2.16) *and* (2.17), *we obtain the following theorem.*

Theorem 2.7. *For* $n \geq 0$, *we have*

$$Ch_{n,q,\frac{1}{2}}(x) = \sum_{m=0}^{n}\binom{n}{l}Ch_{n-l,q,\frac{1}{2}}\sum_{m=0}^{l}\left(\frac{x}{2}\right)^m S_1(l,m).$$

By replacing t *by* $-4t$ *in* (2.16), *we have*

$$\int_{\mathbb{Z}_p}(1-4t)^{\frac{x+y}{2}}d\mu_{-q}(y) = \frac{[2]_q}{1+q\sqrt{1-4t}}\sqrt{(1-4t)^x} = \sum_{n=0}^{\infty} C_{n,q}(x)t^n. \tag{2.18}$$

On the other hand,

$$\sum_{n=0}^{\infty} Ch_{n,q,\frac{1}{2}}(x)\frac{(-4t)^n}{n!} = \sum_{n=0}^{\infty} Ch_{n,q,\frac{1}{2}}(x)(-1)^n 2^{2n}\frac{t^n}{n!}. \tag{2.19}$$

Therefore, by (2.18) *and* (2.19), *we state the following theorem.*

Theorem 2.8. *For* $n \geq 0$, *we have*

$$C_{n,q}(x) = \frac{(-1)^n}{n!}2^{2n}Ch_{n,q,\frac{1}{2}}(x).$$

From (2.16), we note that

$$\frac{[2]_q}{1+q\sqrt{1+t}}\sqrt{(1+t)^x} = \left(\sum_{n=0}^{\infty} Ch_{n,q,\frac{1}{2}}\frac{t^n}{n!}\right)\left(\sum_{m=0}^{\infty}\binom{\frac{x}{2}}{m}(-1)^m 2^{2m} t^m\right)$$

$$= \sum_{n=0}^{\infty}\left(\sum_{m=0}^{n}\binom{\frac{x}{2}}{m}(-1)^m 2^{2m} Ch_{n-m,q,\frac{1}{2}}\frac{1}{(n-m)!}\right)t^n. \tag{2.20}$$

Therefore, by (2.16) and (2.19), we obtain the following theorem.

Theorem 2.9. *For $n \geq 0$, we have*

$$Ch_{n,q,\frac{1}{2}}(x) = \sum_{m=0}^{n}\binom{\frac{x}{2}}{m}(-1)^m 2^{2m} Ch_{n-m,q,\frac{1}{2}}\frac{n!}{(n-m)!}.$$

From (1.2), we see that

$$\sum_{n=0}^{\infty}\int_{\mathbb{Z}_p}(x+y)^n d\mu_{-q}(y)\frac{t^n}{n!} = \int_{\mathbb{Z}_p} e^{(x+y)t} d\mu_{-q}(y) = \frac{[2]_q}{qe^t+1}e^{xt}$$

$$= \sum_{n=0}^{\infty} E_{n,q}(x)\frac{t^n}{n!}, \tag{2.21}$$

where $E_{n,q}(x) = \sum_{m=0}^{n}\binom{n}{m}E_{n-m,q}x^m = \int_{\mathbb{Z}_p}(x+y)^n d\mu_{-q}(y)$ are q-Euler polynomials. From (2.16), we have

$$\frac{[2]_q}{1+q\sqrt{1+t}}\sqrt{(1+t)^x} = \int_{\mathbb{Z}_p}(1+t)^{\frac{x+y}{2}} d\mu_{-q}(y)$$

$$= \sum_{m=0}^{\infty} 2^{-m}\frac{1}{m!}(\log(1+t))^m = \int_{\mathbb{Z}_p}(x+y)^m d\mu_{-q}(y)$$

$$= \sum_{m=0}^{\infty} 2^{-m} E_{m,q}(x)\sum_{n=m}^{\infty} S_1(n,m)\frac{t^n}{n!}$$

$$= \sum_{n=0}^{\infty}\left(\sum_{m=0}^{n} 2^{-m} E_{m,q}(x) S_1(n,m)\right)\frac{t^n}{n!}. \tag{2.22}$$

Thus, by (2.16) and (2.22), we get the following theorem.

Theorem 2.10. *For $n \geq 0$, we have*

$$Ch_{n,q,\frac{1}{2}}(x) = \sum_{m=0}^{n} 2^{-m} E_{m,q}(x) S_1(n,m).$$

3. Conclusion

The aim of the paper is to introduced q-analogue of Catalan numbers $C_{n,q}$ with the help of a p-adic q-integral on $\mathbb{Z}_p$ and derived explicit expressions and some identities for those numbers. In more detail, we deduced explicit expressions of $C_{n,q}$, as a rational function in terms of q-Euler number and Stirling numbers of the first kind, as a fermionic p-adic q-integral on $\mathbb{Z}_p$ and involving q-analogue of $\frac{1}{2}$-Changhee numbers.

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ON FRACTIONAL q - DERIVATIVE INTEGRAL FORMULAE OF PRASAD'S I-FUNCTION I

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ABSTRACT. In the present research work, we have derived two theorems which involves integral operators of Erdélyi-Kober type and a q- analogue of modified multivariable I-function. The related averment for the Riemann-Liouville and Weyl fractional basic integral transforms are also deduced. A number of corollaries concerning the basic analogue of modified multivariable H-function, q - analogue of multivariable H-function and remarks are given at the end of this paper.

1. Introduction

The idea of fractional calculus is considered to have emerged from a question asked by L'Hospital to Leibniz in 1695 [19]. This has obtained more attention during last century because of its vatious specific applications in numerous diverse fields ([15], [16], [17], [31]). The q-calculus was also came in to existence in twentieth century. A detailed theory are given in the books by Slater [35], Exton [8], Gasper [11] and a thesis [7]. The q-extension of the ordinary fractional calculus is known as the fractional q-calculus . In recent times the theory of q-calculus operators have been uesd in several areas. The idea of fractional q-calculus was introduced by Al-Salam. From the basic analogue of Cauchy's formula ([4], [5], [6]), Agarwal [3] studied some fractional basic integral operators and q-derivatives. Later that, Isogawa et al. [13] studied some basic properties of fractional q-derivatives. The notion of the left fractional q-integral operators and fractional q-derivatives was generalized by Rajkovic et al. [21] by introducing variable lower limit and proved the semigroup properties. Garg et al. [10] introduced basic analogues of hyper-Bessel type Kober fractional derivatives. Saxena et al. [33], Yadav et al. ([38]-[43]) have found values of different basic special functions by using fractional q- operators. Inspired by this apporach of applicability, some researchers have applied these integral operators to find value of q - fractional calculus formulae for various special functions. One can see the recent publications [9]-[11] and [33], [22]-[25], [26]-[28], [38]-[43] in this subject.

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In the present research work, we have derived two theorems including the fractional basic integral operator of Erdelyi-Kober type. These theorems generalizes the Riemann-Liouville and Weyl fractional basic integral operators.
For real or complex a and $|q| < 1$, the q-shifted factorial is defined as :

$$(a;q)_n = \prod_{i=1}^{n-1}(1-aq^i) = \frac{(a;q)_\infty}{(aq^n;q)_\infty}, \qquad (n \in \mathbb{N}). \tag{1.1}$$

so that $(a;q)_0 = 1$, or equivalently

$$(a,q)_n = \frac{\Gamma_q(a+n)(1-q)^n}{\Gamma_q(a)} \qquad (a \neq 0,-1,-2,\cdots). \tag{1.2}$$

The basic analogue of Riemann-Liouville operator of a function $f(x)$ by Agarwal [3], is given by

$$I_q^\alpha\{f(x)\} = \frac{1}{\Gamma_q(\alpha)}\int_0^x (x-tq)_{\alpha-1} f(t) d_q t \qquad (Re(\alpha) > 0, |q| < 1). \tag{1.3}$$

The basic analogue of the Kober operator, see Al-Salam [7, 34] is defined by

$$I_q^{\eta,\alpha}\{f(x)\} = \frac{x^{-\eta-\alpha}}{\Gamma_q(\alpha)}\int_0^x (x-tq)_{\alpha-1} t^\eta f(t) d_q t \qquad (Re(\alpha) > 0, , \eta \in \mathbb{R}, |q| < 1). \tag{1.4}$$

A q-analogue of the Weyl integral operator due to Al-Salam [7] is given by

$$K_q^\alpha\{f(x)\} = \frac{q^{(\alpha-1)/2}}{\Gamma_q(\alpha)}\int_x^\infty (t-x)_{\alpha-1} f(tq^{1-\alpha}) d_q t \qquad (Re(\alpha) > 0, |q| < 1). \tag{1.5}$$

Al-Salam [4] defined the following basic analogue:

$$K_q^{\eta,\alpha}\{f(x)\} = \frac{q^{-\eta}x^\eta}{\Gamma_q(\alpha)}\int_x^\infty (t-x)_{\alpha-1} t^{-\eta-\alpha} f(t) d_q t \; (Re(\alpha) > 0, \eta \in \mathbb{R}, |q| < 1). \tag{1.6}$$

The q - integral, see Gasper and Rahman [11] are given by

$$\int_0^x f(t) d_q t = x(1-q)\sum_{k=0}^\infty q^k f(xq^k). \tag{1.7}$$

$$\int_x^\infty f(t) d_q t = x(1-q)\sum_{k=1}^\infty q^{-k} f(xq^{-k}). \tag{1.8}$$

$$\int_0^\infty f(t) d_q t = x(1-q)\sum_{k=-\infty}^\infty q^k f(xq^k). \tag{1.9}$$

2. Base Formula

In this section, we will derive two fractional q-integral formulae for the q - analogue of multivariable I-function defined by Prasad [18]. We note

$$G(q^a) = \left[\prod_{n=0}^\infty (1-q^{a+n})\right]^{-1} = \frac{1}{(q^a;q)_\infty}. \tag{2.1}$$

We have

$$I(z_1,\cdots,z_r;q)=I^{0,n_2;0,n_3;\cdots;0,n_r:m',n';\cdots;m^{(r)},n^{(r)}}_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:p',q';\cdots;p^{(r)},q^{(r)}}\left(\begin{matrix}z_1\\ \vdots\\ z_r\end{matrix};q\;\middle|\;\begin{matrix}(a_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots;\\ (b_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots;\end{matrix}\right.$$

$$\left.\begin{matrix}(a_{rj};\alpha'_{rj},\cdots,\alpha^{(r)}_{rj})_{1,p_r}:(a'_j,\alpha'_j)_{1,p'};\cdots;(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}}\\ (b_{rj};\beta'_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r}:(b'_j,\beta'_j)_{1,q'};\cdots;(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}}\end{matrix}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\pi^r\xi(s_1,\cdots,s_r;q)\prod_{i=1}^{r}\phi_i(s_i;q)t_i^{s_i}ds_1\cdots ds_r, \tag{2.2}$$

where

$$\xi(s_1,\cdots,s_r;q)=\frac{\prod_{j=1}^{n_2}G(q^{1-a_{2j}+\sum_{i=1}^{2}\alpha^{(i)}_{2j}\xi})\prod_{j=1}^{n_3}G(q^{1-a_{3j}+\sum_{i=1}^{3}\alpha^{(i)}_{3j}\xi})\cdots}{\prod_{j=n_2+1}^{p_2}G(q^{a_{2j}-\sum_{i=1}^{2}\alpha^{(i)}_{2j}\xi})\prod_{j=n_3+1}^{p_3}G(q^{a_{3j}-\sum_{i=1}^{3}\alpha^{(i)}_{3j}\xi})\cdots}$$

$$\frac{\cdots\prod_{j=1}^{n_r}G(q^{1-a_{rj}+\sum_{i=1}^{r}\alpha^{(i)}_{rj}s_i})}{\cdots\prod_{n_r+1}^{p_r}G(q^{a_{rj}-\sum_{i=1}^{r}\alpha^{(i)}_{rj}s_i})\prod_{j=1}^{q_2}G(q^{1-b_{2j}-\sum_{i=1}^{2}\beta^{(i)}_{2j}s_i})\cdots\prod_{j=1}^{q_r}G(q^{1-b_{rj}-\sum_{i=1}^{r}\beta^{(i)}_{rj}s_i})}. \tag{2.3}$$

$$\phi(s_i,q)=\frac{\prod_{j=1}^{m^{(i)}}G(q^{b^{(i)}_j-\beta^{(i)}_j s})\prod_{j=1}^{n^{(i)}}G(q^{1-a^{(i)}_j+\alpha^{(i)}_j s_i})}{\prod_{j=1+m^{(i)}}^{q^{(i)}}G(q^{1-b^{(i)}_j+\beta^{(i)}_j s_i})\prod_{j=n^{(i)}+1}^{p^{(i)}}G(q^{a^{(i)}_j-\alpha^{(i)}_j s_i})G(1-q^{s_i})\sin\pi s_i}, \tag{2.4}$$

$\alpha^{(i)}_j,\beta^{(i)}_j,\alpha^{(i)}_{kj},\beta^{(i)}_{kj}(i=1,\cdots,r),(k=1,\cdots,r)$ are positive numbers.
$a^{(i)}_j,b^{(i)}_j(i=1,\cdots,r),\alpha_{kj},\beta_{kj}(k=2,\cdots,r)$ are complex numbers and here $m^{(i)}$, $n^{(i)}$, $p^{(i)}$, $q^{(i)}$ $(i=1,\cdots,r)$, n_k, p_k, $q_k(k=2,\cdots,r)$ are non-negative integers where $0\le m^{(i)}\le q^{(i)};0\le n^{(i)}\le p^{(i)}(i=1,\cdots,r),0\le q^{(k)}$ and $0\le n_k\le p_k$.
Here (i) denotes the numbers of dashes. The contour L_k is in the $s_k(k=1,\cdots,r)$-plane and varies from $\sigma-i\infty$ to $\sigma+i\infty$ where σ if is a real number, if necessary to ensure that the poles of $G(q^{1-a_{2j}+\sum_{k=1}^{2}\alpha^{(k)}_{2j}s_k}),(j=1,\cdots,n_2),G(q^{1-a_{3j}+\sum_{k=1}^{3}\alpha^{(k)}_{3j}s_k})$, $(j=1,\cdots,n_3),G(q^{1-a_{rj}+\sum_{k=1}^{r}\alpha^{(k)}_{rj}s_k})$, $(j=1,\cdots,n_r)$, $G(q^{1-c^{(k)}_j+\gamma^{(k)}_j s_k}),(j=1,\cdots,n^{(k)}),(k=1,\cdots,r)$ to the left of the contour L_k and the poles of $G(q^{d^{(k)}_j-\delta^{(k)}_j s_k})$, $(j=1,\cdots,m^{(k)}),(k=1,\cdots,r)$ lie to the right of the contourL_k. For further details and asymptotic expansion of the I-function one can refer by Prasad [18]. For large values of $|s_i|$ $\;Re(s_i log(z_i)-log\sin\pi s_i)<0$, $i=1,\cdots,r$. It is assumed that integrand function has simple poles.
We note

$$A=(a_{2j};\alpha'_{2j},\alpha''_{2j})_{1,p_2};\cdots;(a_{(r-1)j};\alpha'_{(r-1)j},\cdots,\alpha^{r-1}_{(r-1)j})_{1,p_{r-1}}. \tag{2.5}$$

$$B=(b_{2j};\beta'_{2j},\beta''_{2j})_{1,q_2};\cdots;(b_{(r-1)j};\beta'_{(r-1)j},\cdots,\beta^{r-1}_{(r-1)j})_{1,q_{r-1}}. \tag{2.6}$$

$$\mathbb{A}=(a_{rj};\alpha'_{rj},\cdots,\alpha^{(r)}_{rj})_{1,p_r}:\mathfrak{A}=(a'_j,\alpha'_j)_{1,p'};\cdots;(a^{(r)}_j,\alpha^{(r)}_j)_{1,p^{(r)}}. \tag{2.7}$$

$$\mathbb{B}=(b_{rj};\beta'_{rj},\cdots,\beta^{(r)}_{rj})_{1,q_r}:\mathfrak{B}=(b'_j,\beta'_j)_{1,q'};\cdots;(b^{(r)}_j,\beta^{(r)}_j)_{1,q^{(r)}}. \tag{2.8}$$

$$U=p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1};\;V=0,n_2;0,n_3;\cdots;0,n_{s-1}. \tag{2.9}$$

$$W=(p',q');\cdots;(p^{(r)},q^{(r)});X=(m',n');\cdots;(m^{(r)},n^{(r)}). \tag{2.10}$$

3. Results

We shall derive two fractional basic integral formulae for the q- analogue of multivariable I-function.

Theorem 3.1. *Let $Re(\mu) > 0, |q| < 1, \eta \in \mathbb{R}, \rho_i > 0 (i = 1, \cdots, r)$ and $I_q^{\eta,\alpha}\{.\}$ be the Kober fractional q-integral operator (1.4), then the following result holds :*

$$I_q^{\eta,\mu}\left\{ x^{\lambda-1} I\left(\begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ; q \middle| \begin{array}{c} A; \mathbb{A} : \mathfrak{A}; \\ \\ B; \mathbb{B} : \mathfrak{B} \end{array} \right) \right\} = (1-q)^{\mu} x^{\lambda-1}$$

$$I_{U;p_r+1,q_r+1:W}^{V;0,n_r+1:X}\left(\begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ; q \middle| \begin{array}{c} A; (1-\lambda-\eta; \rho_1, \cdots, \rho_r), \mathbb{A} : \mathfrak{A} \\ \\ B; \mathbb{B}(1-\lambda-\mu-\eta; \rho_1, \cdots, \rho_r) : \mathfrak{B} \end{array} \right), \quad (3.1)$$

where $Re(s_i \log(z_i) - \log\sin \pi s_i) < 0, (i = 1, \cdots, r)$.

Proof. To prove this theorem, we use equation (3.1) (say I) and using the definitions (1.4) and (2.2), we get

$$I = \frac{x^{-\eta-\alpha}}{\Gamma_q(\alpha)} \int_0^x (x - yq)_{\alpha-1} y^{\eta} \left\{ y^{\lambda-1} \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \pi^r \xi(s_1, \cdots, s_r; q) \right.$$

$$\left. \prod_{i=1}^{r} \phi_k(s_{i_r}; q)(z_i y^{\rho_i})^{s_i} d_q s_1 \cdots d_q s_r \right\} = \frac{x^{-\eta-\alpha}}{(2\pi\omega)^r \Gamma_q(\alpha)} \int_{L_1} \cdots \int_{L_r} \pi^r \xi(s_1, \cdots, s_r; q)$$

$$\prod_{i=1}^{r} \phi_k(s_i; q) z_i^{s_i} \left\{ \int_0^x (x - yq)_{\alpha-1} y^{\eta} y^{\lambda + \sum_{i=1}^{r} \rho_i s_i - 1} d_q y \right\} d_q s_1 \cdots d_q s_r$$

$$= \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \pi^r \xi(s_1, \cdots, s_r; q) \prod_{i=1}^{r} \theta_k(s_i; q) z_i^{s_i} I_q^{\eta,\mu}\{x^{\lambda + \sum_{i=1}^{r} \rho_i s_i - 1}\} d_q s_1 \cdots d_q s_r$$

Applying the formula by Yadav and Purohit ([39], p. 440, eq. (19))

$$I_q^{\eta,\mu}\{x^{\lambda-1}\} = \frac{\Gamma_q(\lambda+\eta)}{\Gamma_q(\lambda+\eta+\mu)} x^{\eta+\lambda-1}, \ (Re(\lambda+\mu) > 0). \quad (3.2)$$

Substituting (3.2) in the above equation, we obtain

$$\frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \pi^r \xi(s_1, \cdots, s_r; q) \prod_{i=1}^{r} \theta_k(s_i; q) z_i^{s_i}$$

$$\frac{\Gamma_q(\lambda + \eta + \sum_{i=1}^{r} \rho_i s_i)}{\Gamma_q(\lambda + \eta + \mu + \sum_{i=1}^{r} \rho_i s_i)} x^{\lambda + \eta + \sum_{i=1}^{r} \rho_i s_i - 1} d_q s_1 \cdots d_q s_r. \quad (3.3)$$

Now, deducing the q- Mellin- Barnes double contour integrals in terms of the q - analogue of Aleph- function of several variables, the required result (3.1) can be obtained. □

For $Re(\mu) > 0, |q| < 1, \ \rho_i (i = 1, \cdots, r)$ positive integers, the Riemann Liouville fractional q-integral of the product of two basic functions exists.

Theorem 3.2. *Let $Re(\mu) > 0, |q| < 1, \eta \in R, \rho_i > 0 (i = 1, \cdots, r)$ then the generalized Weyl q-integral operator for the basic analogue multivariable I-function is given by*

$$K_q^{\eta,\mu}\left\{ x^{\lambda-1} I \left(\begin{array}{c} z_1 x^{-\rho_1} \\ \vdots \\ z_r x^{-\rho_r} \end{array} ;q \left| \begin{array}{c} A; \mathbb{A} : \mathfrak{A}; \\ \\ B; \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = x^{\lambda}(1-q)^{\mu} q^{-\mu\lambda}$$

$$I^{V;0,n_r+1:X}_{U;p_r+1,q_r+1:W} \left(\begin{array}{c} z_1 (xq^{-\mu})^{-\rho_1} \\ \vdots \\ z_r (xq^{-\mu})^{-\rho_r} \end{array} ;q \left| \begin{array}{c} A; (1+\lambda-\eta; \rho_1, \cdots, \rho_r), \mathbb{A} : \mathfrak{A}; \\ \\ B; \mathbb{B}(1+\lambda-\mu-\eta; \rho_1, \cdots, \rho_r) : \mathfrak{B} \end{array} \right. \right), \tag{3.4}$$

where $Re(\xi_i log(z_i) - log \sin \pi \xi_i) < 0, (i = 1, \cdots, r)$.

Proof. To prove this result we apply equation (1.6) in equation (3.4). Then we write in integral form using equation (2.2). After that using equation (3.2) we can obtain the required theorem. □

4. Special Cases:

Corollary 4.1. *Let $Re(\mu) > 0, |q| < 1, \eta \in R$ and $I_q^{\mu}\{.\}$ be the Riemann-Liouville fractional q-integral operator (1.3), then the following result holds :*

$$I_q^{\mu}\left\{ x^{\lambda-1} I \left(\begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ;q \left| \begin{array}{c} A; \mathbb{A} : \mathfrak{A} \\ \\ B; \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = (1-q)^{\mu} x^{\lambda+\mu-1}$$

$$I^{V;0,n_r+1:X}_{U;p_r+1,q_r+1:W} \left(\begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ;q \left| \begin{array}{c} A; (1-\lambda; \rho_1, \cdots, \rho_r), \mathbb{A} : \mathfrak{A}; \\ \\ B; \mathbb{B}(1-\lambda-\mu; \rho_1, \cdots, \rho_r) : \mathfrak{B} \end{array} \right. \right), \tag{4.1}$$

where $Re(s_i log(z_i) - log \sin \pi s_i) < 0, (i = 1, \cdots, r)$.

Corollary 4.2. *Let $Re(\mu) > 0, |q| < 1, \eta \in R$ and $K_q^{\mu}\{.\}$ be the Weyl fractional q - integral operator (1.5) then the following result holds:*

$$K_q^{\mu}\left\{ x^{\lambda-1} I \left(\begin{array}{c} z_1 x^{-\rho_1} \\ \vdots \\ z_r x^{-\rho_r} \end{array} ;q \left| \begin{array}{c} A; \mathbb{A} : \mathfrak{A} \\ \\ B; \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = x^{\mu+\lambda}(1-q)^{\mu} q^{-\mu\lambda-\mu(\mu+1)/2}$$

$$I^{V;0,n_r+1:X}_{U;p_r+1,q_r+1:W} \left(\begin{array}{c} z_1 (xq^{-\mu})^{-\rho_1} \\ \vdots \\ z_r (xq^{-\mu})^{-\rho_r} \end{array} ;q \left| \begin{array}{c} A; (1+\lambda+\mu; \rho_1, \cdots, \rho_r), \mathbb{A} : \mathfrak{A} \\ \\ B; \mathbb{B}(1+\lambda; \rho_1, \cdots, \rho_r) : \mathfrak{B} \end{array} \right. \right), \tag{4.2}$$

where $Re(s_i log(z_i) - log \sin \pi s_i) < 0, (i = 1, \cdots, r)$.

Theorem 3.2. *Let $Re(\mu) > 0, |q| < 1, \eta \in R$, $\rho_i > 0 (i = 1, \cdots, r)$ then the generalized Weyl q-integral operator for the basic analogue multivariable I-function is given by*

$$K_q^{\eta,\mu}\left\{ x^{\lambda-1} I \left(\begin{array}{c} z_1 x^{-\rho_1} \\ \vdots \\ z_r x^{-\rho_r} \end{array} ;q \left| \begin{array}{c} A; \mathbb{A} : \mathfrak{A}; \\ \\ B; \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = x^{\lambda}(1-q)^{\mu} q^{-\mu\lambda}$$

$$I^{V;0,n_r+1:X}_{U;p_r+1,q_r+1:W} \left(\begin{array}{c} z_1 (xq^{-\mu})^{-\rho_1} \\ \vdots \\ z_r (xq^{-\mu})^{-\rho_r} \end{array} ;q \left| \begin{array}{c} A; (1+\lambda-\eta; \rho_1, \cdots, \rho_r), \mathbb{A} : \mathfrak{A}; \\ \\ B; \mathbb{B}(1+\lambda-\mu-\eta; \rho_1, \cdots, \rho_r) : \mathfrak{B} \end{array} \right. \right), \tag{3.4}$$

where $Re(\xi_i log(z_i) - log \sin \pi \xi_i) < 0, (i = 1, \cdots, r)$.

Proof. To prove this result we apply equation (1.6) in equation (3.4). Then we write in integral form using equation (2.2). After that using equation (3.2) we can obtain the required theorem. □

4. Special Cases:

Corollary 4.1. *Let $Re(\mu) > 0, |q| < 1, \eta \in R$ and $I_q^{\mu}\{.\}$ be the Riemann-Liouville fractional q-integral operator (1.3), then the following result holds :*

$$I_q^{\mu}\left\{ x^{\lambda-1} I \left(\begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ;q \left| \begin{array}{c} A; \mathbb{A} : \mathfrak{A} \\ \\ B; \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = (1-q)^{\mu} x^{\lambda+\mu-1}$$

$$I^{V;0,n_r+1:X}_{U;p_r+1,q_r+1:W} \left(\begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ;q \left| \begin{array}{c} A; (1-\lambda; \rho_1, \cdots, \rho_r), \mathbb{A} : \mathfrak{A}; \\ \\ B; \mathbb{B}(1-\lambda-\mu; \rho_1, \cdots, \rho_r) : \mathfrak{B} \end{array} \right. \right), \tag{4.1}$$

where $Re(s_i log(z_i) - log \sin \pi s_i) < 0, (i = 1, \cdots, r)$.

Corollary 4.2. *Let $Re(\mu) > 0, |q| < 1, \eta \in R$ and $K_q^{\mu}\{.\}$ be the Weyl fractional q - integral operator (1.5) then the following result holds:*

$$K_q^{\mu}\left\{ x^{\lambda-1} I \left(\begin{array}{c} z_1 x^{-\rho_1} \\ \vdots \\ z_r x^{-\rho_r} \end{array} ;q \left| \begin{array}{c} A; \mathbb{A} : \mathfrak{A} \\ \\ B; \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = x^{\mu+\lambda}(1-q)^{\mu} q^{-\mu\lambda-\mu(\mu+1)/2}$$

$$I^{V;0,n_r+1:X}_{U;p_r+1,q_r+1:W} \left(\begin{array}{c} z_1 (xq^{-\mu})^{-\rho_1} \\ \vdots \\ z_r (xq^{-\mu})^{-\rho_r} \end{array} ;q \left| \begin{array}{c} A; (1+\lambda+\mu; \rho_1, \cdots, \rho_r), \mathbb{A} : \mathfrak{A} \\ \\ B; \mathbb{B}(1+\lambda; \rho_1, \cdots, \rho_r) : \mathfrak{B} \end{array} \right. \right), \tag{4.2}$$

where $Re(s_i log(z_i) - log \sin \pi s_i) < 0, (i = 1, \cdots, r)$.

5. Basic of Multivariable H- Function

Setting $A = B = 0 = U = V$, q - analogue I of several variables converts in q-analogue of H-function of several variables defined by Prasad and Singh [19]. We obtain

Corollary 5.1.

$$I_q^{\eta,\mu}\left\{ x^{\lambda-1} H\left(\begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ;q \left| \begin{array}{c} \mathbb{A} : \mathfrak{A} \\ \\ \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = (1-q)^{\mu} x^{\lambda-1}$$

$$H^{0,n_r+1:X}_{p_r+1,q_r+1:W}\left(\begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ;q \left| \begin{array}{c} (1-\lambda-\eta;\rho_1,\cdots,\rho_r), \mathbb{A} : \mathfrak{A} \\ \\ \mathbb{B}; (1-\lambda-\mu-\eta;\rho_1,\cdots,\rho_r) : \mathfrak{B} \end{array} \right. \right), \quad (5.1)$$

where $Re(s_i log(z_i) - log \sin \pi s_i) < 0, (i = 1, \cdots, r)$.

Corollary 5.2.

$$K_q^{\eta,\mu}\left\{ x^{\lambda-1} H\left(\begin{array}{c} z_1 x^{-\rho_1} \\ \vdots \\ z_r x^{-\rho_r} \end{array} ;q \left| \begin{array}{c} \mathbb{A} : \mathfrak{A} \\ \\ \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = x^{\lambda}(1-q)^{\mu} q^{-\mu\lambda}$$

$$H^{0,n_r+1:X}_{p_r+1,q_r:W}\left(\begin{array}{c} z_1 (xq^{-\mu})^{-\rho_1} \\ \vdots \\ z_r (xq^{-\mu})^{-\rho_r} \end{array} ;q \left| \begin{array}{c} (1+\lambda-\eta;\rho_1,\cdots,\rho_r), \mathbb{A} : \mathfrak{A} \\ \\ \mathbb{B}; (1+\lambda-\mu-\eta;\rho_1,\cdots,\rho_r) : \mathfrak{B} \end{array} \right. \right), \quad (5.2)$$

where $Re(\xi_i log(z_i) - log \sin \pi \xi_i) < 0, (i = 1, \cdots, r)$.

Corollary 5.3.

$$I_q^{\mu}\left\{ x^{\lambda-1} H\left(\begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ;q \left| \begin{array}{c} \mathbb{A} : \mathfrak{A} \\ \\ \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = (1-q)^{\mu} x^{\lambda+\mu-1}$$

$$H^{0,n_r+1:X}_{p_r+1,q_r+1:W}\left(\begin{array}{c} z_1 x^{\rho_1} \\ \vdots \\ z_r x^{\rho_r} \end{array} ;q \left| \begin{array}{c} (1-\lambda;\rho_1,\cdots,\rho_r), \mathbb{A} : \mathfrak{A} \\ \\ \mathbb{B}; (1-\lambda-\mu;\rho_1,\cdots,\rho_r) : \mathfrak{B} \end{array} \right. \right), \quad (5.3)$$

where $Re(s_i log(z_i) - log \sin \pi s_i) < 0, (i = 1, \cdots, r)$.

Corollary 5.4.

$$K_q^{\mu}\left\{ x^{\lambda-1} H\left(\begin{array}{c} z_1 x^{-\rho_1} \\ \vdots \\ z_r x^{-\rho_r} \end{array} ;q \left| \begin{array}{c} \mathbb{A} : \mathfrak{A} \\ \\ \mathbb{B} : \mathfrak{B} \end{array} \right. \right) \right\} = x^{\mu+\lambda}(1-q)^{\mu} q^{-\mu\lambda-\mu(\mu+1)/2}$$

$$H^{0,n_r+1:X}_{p_r+1,q_r+1:W}\left(\begin{array}{c} z_1 (xq^{-\mu})^{-\rho_1} \\ \vdots \\ z_r (xq^{-\mu})^{-\rho_r} \end{array} ;q \left| \begin{array}{c} (1+\lambda+\mu;\rho_1,\cdots,\rho_r), \mathbb{A} : \mathfrak{A} \\ \\ \mathbb{B}; (1+\lambda;\rho_1,\cdots,\rho_r) : \mathfrak{B} \end{array} \right. \right), \quad (5.4)$$

where $Re(s_i log(z_i) - log \sin \pi s_i) < 0, (i = 1, \cdots, r)$.

Taking $r=2$, then the q - analogue of H - function of several variables reduces to q - analogue of H - function of two variables [12] given by Saxena et al.[33], Let

$$C_2=\{(a_i;\alpha_i,A_i)\}_{1,p_1}D_2=\{(e_i;E_i)\}_{1,p_2},\{(g_i;G_i)\}_{1,p_3}. \tag{5.5}$$

$$E_2=\{(b_i;\beta_i,B_i)\}_{1,q_1}F_2=\{(f_i;F_i)\}_{1,q_2},\{(h_i;H_i)\}_{1,q_3}. \tag{5.6}$$

We get:

Corollary 5.5.

$$I_q^{\eta,\mu}\left\{x^{\lambda-1}H^{m_1,n_1;m_2,n_2,m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left(\begin{array}{c|c}\begin{matrix}z_1x^{\rho}\\ \vdots\\ z_2x^{\sigma}\end{matrix};q & \begin{matrix}C_2:D_2\\ \\ E_2:F_2\end{matrix}\end{array}\right)\right\}=(1-q)^{\mu}x^{\lambda-1}$$

$$H^{m_1,n_1+1;m_2,n_2,m_3,n_3}_{p_1+1,q_1+1;p_2,q_2;p_3,q_3}\left(\begin{array}{c|c}\begin{matrix}z_1x^{\rho}\\ \vdots\\ z_2x^{\sigma}\end{matrix};q & \begin{matrix}(1-\lambda-\eta;\rho,\sigma),C_2:D_2\\ \\ E_2,(1-\lambda-\mu-\eta;\rho,\sigma):F_2\end{matrix}\end{array}\right), \tag{5.7}$$

under the conditions verified by the corollary 5.1 and $r=2$, see Yadav et al. [42].

Corollary 5.6.

$$K_q^{\eta,\mu}\left\{x^{\lambda-1}H^{m_1,n_1;m_2,n_2,m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left(\begin{array}{c|c}\begin{matrix}z_1x^{-\rho}\\ \vdots\\ z_2x^{-\sigma}\end{matrix};q & \begin{matrix}C_2:D_2\\ \\ E_2:F_2\end{matrix}\end{array}\right)\right\}=x^{\lambda}(1-q)^{\mu}q^{-\mu\lambda}$$

$$H^{m_1+1,n_1;m_2,n_2,m_3,n_3}_{p_1+1,q_1+1;p_2,q_2;p_3,q_3}\left(\begin{array}{c|c}\begin{matrix}z_1(xq^{-\mu})^{-\rho}\\ \vdots\\ z_2(xq^{-\mu})^{-\sigma}\end{matrix};q & \begin{matrix}(1+\lambda-\eta;\rho,\sigma),C_2:D_2\\ \\ E_2,(1+\lambda-\mu-\eta;\rho,\sigma):F_2\end{matrix}\end{array}\right), \tag{5.8}$$

under the conditions verified by the corollary 5.2 and $r=2$, see Yadav et al. [42].

We suppose

$$(\alpha_j)_{1,p_1}=(A_j)_{1,p_1}=(E_j)_{1,p_2}=(G_j)_{1,p_3}=(\beta_j)_{1,q_1}=(B_j)_{1,q_1}=(F_j)_{1,q_2}=(H_j)_{1,q_3}=1. \tag{5.9}$$

The q - analogue of Meijer's G - function of two variables given by Agarwal [2], can also be obtained by setting the following parameters:

$$A_2'=(a_j)_{1,p_1}:B_2'=(e_j)_{1,p_2},(g_j)_{1,p_3}. \tag{5.10}$$

$$C_2'=(b_j)_{1,q_1}:D_2'=(f_j)_{1,q_2},(h_j)_{1,q_3}. \tag{5.11}$$

and we obtain the formulas as follows:

Corollary 5.7.

$$I_q^{\eta,\mu}\left\{x^{\lambda-1}\left(\begin{array}{c|c}\begin{matrix}z_1x^{\rho}\\ \vdots\\ z_2x^{\sigma}\end{matrix};q & \begin{matrix}A_2':C_2'\\ \\ B_2':D_2'\end{matrix}\end{array}\right)\right\}=(1-q)^{\mu}x^{\lambda-1}$$

$$G^{m_1,n_1+1;m_2,n_2,m_3,n_3}_{p_1+1,q_1+1;p_2,q_2;p_3,q_3}\left(\begin{array}{c|c}\begin{matrix}z_1x^{\rho}\\ \vdots\\ z_2x^{\sigma}\end{matrix};q & \begin{matrix}(1-\lambda-\eta;\rho,\sigma),C_2':D_2'\\ \\ E_2',(1-\lambda-\mu-\eta;\rho,\sigma):F_2'\end{matrix}\end{array}\right), \tag{5.12}$$

with the conditions verified by the corollary 5.7 and $(\alpha_j)_{1,p_1} = (A_j)_{1,p_1} = (E_j)_{1,p_2} = (G_j)_{1,p_3} = (\beta_j)_{1,q_1} = (B_j)_{1,q_1} = (F_j)_{1,q_2} = (H_j)_{1,q_3} = 1.$

Corollary 5.8.

$$K_q^{\eta,\mu}\left\{ x^{\lambda-1} G^{0,n_1;m_2,n_2,m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left(\begin{array}{c} z_1 x^{-\rho} \\ \vdots \\ z_2 x^{-\sigma} \end{array} ;(p,q) \left| \begin{array}{c} A_2' : B_2' \\ \\ C_2' : D_2' \end{array} \right. \right) \right\} = x^{\lambda}(1-q)^{\mu} q^{-\mu\lambda}$$

$$G^{0,n_1+1;m_2,n_2,m_3,n_3}_{p_1+1,q_1+1;p_2,q_2;p_3,q_3}\left(\begin{array}{c} z_1(xq^{-\mu})^{-\rho} \\ \vdots \\ z_2(xq^{-\mu})^{-\sigma} \end{array} ;q \left| \begin{array}{c} (1+\lambda-\eta;\rho,\sigma), A_2' : B_2' \\ \\ C_2', (1+\lambda-\mu-\eta;\rho,\sigma) : D_2' \end{array} \right. \right), \tag{5.13}$$

with the conditions verified by the corollary 5.6 and $(\alpha_j)_{1,p_1} = (A_j)_{1,p_1} = (E_j)_{1,p_2} = (G_j)_{1,p_3} = (\beta_j)_{1,q_1} = (B_j)_{1,q_1} = (F_j)_{1,q_2} = (H_j)_{1,q_3} = 1.$

If $r = 1$, the q - analogue of multivariable I - function converts in q - analogue of I - function defined by Rathie [29].
Let $A_1 = (a_j, \alpha_j : A_j)_{1,p}; B_1 = (b_j, \beta_j : B_j)_{1,q}$, we obtain the following result

Corollary 5.9.

$$I_q^{\eta,\mu}\left\{ x^{\lambda-1} I^{m,n}_{p,q'}\left(zx^{\rho} \; ;q \left| \begin{array}{c} A_1 \\ \cdot \\ B_1 \end{array} \right. \right) \right\} = (1-q)^{\mu} x^{\lambda-1}$$

$$I^{m,n+1}_{p+1,q'+1}\left(zx^{\rho} \; ;q \left| \begin{array}{c} (1-\lambda-\eta;\rho;1), A_1 \\ \\ B_1, (1-\lambda-\mu-\eta;\rho;1) \end{array} \right. \right), \tag{5.14}$$

where $Re(slog(z_1) - log \sin \pi s) < 0.$

Corollary 5.10.

$$K_q^{\eta,\mu}\left\{ x^{\lambda-1} I^{m,n}_{p,q'}\left(zx^{-\rho} \; ;q \left| \begin{array}{c} A_1 \\ \cdot \\ B_1 \end{array} \right. \right) \right\} = x^{\lambda}(1-q)^{\mu} q^{-\mu\lambda}$$

$$I^{m,n+1}_{p+1,q'+1}\left(z(xq^{-\mu})^{-\rho} \; ;q \left| \begin{array}{c} (1+\lambda-\eta;\rho;1), A_1 \\ \\ B_1, (1+\lambda-\mu-\eta;\rho;1) \end{array} \right. \right), \tag{5.15}$$

where $Re(slog(z_1) - log \sin \pi s) < 0.$

Let $A_1' = (a_j, \alpha_j)_{1,p}$ and $B_1' = (b_j, \beta_j)_{1,q}$, the q - analogue of I - function changes to q - analogue of H - function of one variable defined by Saxena et. al.[32]. We have the following results:

Corollary 5.11.

$$I_q^{\eta,\mu}\left\{ x^{\lambda-1} H^{m,n}_{p,q}\left(zx^{\rho} \; ;q \left| \begin{array}{c} A_1' \\ B_1' \end{array} \right. \right) \right\}$$

$$= (1-q)^{\mu} x^{\lambda-1} H^{m,n+1}_{p+1,q+1}\left(zx^{\rho} \; ;q \left| \begin{array}{c} (1-\lambda-\eta;\rho), A_1' \\ B_1', (1-\lambda-\mu-\eta;\rho) \end{array} \right. \right), \tag{5.16}$$

under the condition verified by the corollary 5.9 and $(A_j)_{1,p} = (B_j)_{1,q} = 1.$

Corollary 5.12.

$$K_q^{\eta,\mu}\left\{ x^{\lambda-1} H_{p,q}^{m,n}\left(zx^{-\rho} \;;q\middle|\; \begin{matrix} A_1^{'} \\ B_1^{'}\end{matrix}\right)\right\} = x^{\lambda}(1-q)^{\mu} q^{-\mu\lambda}$$

$$H_{p+1,q+1}^{m,n+1}\left(z(xq^{-\mu})^{-\rho} \;;q\middle|\; \begin{matrix}(1+\lambda-\eta;\rho),A_1^{'}\\ B_1^{'},(1+\lambda-\mu-\eta;\rho)\end{matrix}\right), \tag{5.17}$$

under the condition verified by the corollary 5.10 and $(A_j)_{1,p} = (B_j)_{1,q} = 1$.

Let $A_1^{''} = (a_j)_{1,p}$ and $B_1^{''} = (b_j)_{1,q}$, the basic of H- function of one variable reduce to basic of Meijer's G-functions, this gives:

Corollary 5.13.

$$I_q^{\eta,\mu}\left\{ x^{\lambda-1} G_{p,q}^{m,n}\left(zx^{\rho} \;;q\middle|\; \begin{matrix} A_1^{''} \\ B_1^{''}\end{matrix}\right)\right\}$$

$$= (1-q)^{\mu} x^{\lambda-1} G_{p+1,q+1}^{m,n+1}\left(zx^{\rho} \;;q\middle|\; \begin{matrix}(1-\lambda-\eta;\rho),A_1^{''}\\ B_1^{''},(1-\lambda-\mu-\eta;\rho)\end{matrix}\right), \tag{5.18}$$

under the condition verified by the corollary 5.11 and $(\alpha_j)_{1,p} = (\beta_j)_{1,q} = 1$.

Corollary 5.14.

$$K_q^{\eta,\mu}\left\{ x^{\lambda-1} G_{p,q}^{m,n}\left(zx^{-\rho} \;;q\middle|\; \begin{matrix} A_1^{''} \\ B_1^{''}\end{matrix}\right)\right\} = x^{\lambda}(1-q)^{\mu} q^{-\mu\lambda}$$

$$G_{p+1,q+1}^{m,n+1}\left(z(xq^{-\mu})^{-\rho} \;;q\middle|\; \begin{matrix}(1+\lambda-\eta;\rho),A_1^{''}\\ B_1^{''},(1+\lambda-\mu-\eta;\rho)\end{matrix}\right), \tag{5.19}$$

under the condition verified by the corollary 5.12 and $(\alpha_j)_{1,p} = (\beta_j)_{1,q} = 1$.

6. Conclusion

The results obtained in this research paper have various applications due to its general nature. By putting some particular values of the parameters and variables in the q- analogue of multivariable I-function, we can deduce various results. These results include various type of basic functions and can be expressed in terms of q-analogue of H-function [32], q- analogue of Meijer's G-function, q- analogue of E-function, q- analogue of hypergeometric function of one variable and multivariable [43], special basic functions of one and several variables. The results proved in this research paper are generalised results of multivariable I - functions. Therefore a number of results in literature can be expressed in terms of these results.

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ON CERTAIN IDENTITIES INVOLVING BASIC (q) HYPERGEOMETRIC SERIES

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Abstract: In this paper we establish certain identities by making use of Bailey's ${}_2\Psi_2$ transformation formula. Special cases of these identities have also been discussed.

Keywords and Phrases: Transformation formula, identity, basic bilateral hypergeometric series, summation formula.

2020 Mathematics Subject Classification: 33C15, 33C20, 33D15.

1. Introduction, Notations and Definitions

Throughout the present paper, we adopt the following notations and definitions. For a and q complex numbers with $|q| < 1$ the q-shifted factorial is defined as,

$$(a;q)_n = \frac{(a;q)_\infty}{(aq^n;q)_\infty} = (1-a)(1-aq)...(1-aq^{n-1}),$$

$$(a;q)_0 = 1$$

and

$$(a;q)_\infty = \prod_{r=0}^{\infty}(1-aq^r).$$

For brevity we write,

$$(a_1;q)_n(a_2;q)_n...(a_r;q)_n = (a_1,a_2,...,a_r;q)_n.$$

Also,

$$(a;q)_{-n} = \frac{(-1)^n q^{n(n+1)/2}}{a^n(q/a;q)_n}.$$

Following [Gasper and Rahman [5]] the basic hypergeometric series is defined as,

$$ {}_r\Phi_s\left[\begin{array}{l} a_1,a_2,...,a_r;q;z \\ b_1,b_2,...,b_s \end{array}\right] = \sum_{n=0}^{\infty}\frac{(a_1,a_2,...,a_r;q)_n}{(q,b_1,b_2,...,b_s;q)_n}z^n\left\{(-1)^n q^{n(n-1)/2}\right\}^{1+s-r}, \quad (1.1)$$

which converges for $|z|<\infty$ if $r\le s$ and for $|z|<1$ if $r=s+1$.
The basic bilateral hypergeometric series is defined as

$$ {}_r\Psi_s\left[\begin{array}{l} a_1,a_2,...,a_r;q;z \\ b_1,b_2,...,b_s \end{array}\right] = \sum_{n=-\infty}^{\infty}\frac{(a_1,a_2,...,a_r;q)_n}{(b_1,b_2,...,b_s;q)_n}z^n\left\{(-1)^n q^{n(n-1)/2}\right\}^{s-r}, \quad (1.2)$$

which converges for $\left|\frac{b_1b_2...b_s}{a_1a_2...a_r}\right| < |z| < 1$ if $r=s$ and for $s>r$ it converges in the whole complex-plane i.e. for $|z|<\infty$.
A great deal of literature is available on special functions of two and more variables, transformations formulas and identities [1, 2, 5]. However, the literature on basic multiple hypergeometric functions seems to be a lot less extensive. Apart from the aforementioned work on basic (q) series identities have developed [6, 7, 8, 9, 10, 11] various interesting properties of basic (q) series and their generalizations and special cases. In the present paper we prove a number of general bilateral q-series identities and transformations which are shown to be applicable in the derivation of continued fraction and partition theoretic interpretation and its generating functions. We also consider several other interesting consequences of some of the results presented here.

2. Main Results

In this section we establish following results

$$(b-c)\ {}_3\Psi_3\left[\begin{array}{l} a, b, c; q; \frac{1}{a} \\ d, bq, cq \end{array}\right] = \frac{(q;q)_\infty^2}{(q/a, d; q)_\infty}$$
$$\times\left\{b^2(1-c)\frac{(bq/a, d/b; q)_\infty}{(bq, q/b; q)_\infty} - c^2(1-b)\frac{(cq/a, d/c; q)_\infty}{(cq, q/c; q)_\infty}\right\}. \qquad (2.1)$$

$$(b-c)\ {}_3\Psi_3\left[\begin{array}{l} a, b, c; q; \frac{q}{a} \\ d, bq, cq \end{array}\right] = \frac{(q;q)_\infty^2}{(q/a, d; q)_\infty}$$
$$\times\left\{b(1-c)\frac{(bq/a, d/b; q)_\infty}{(bq, q/b; q)_\infty} - c(1-b)\frac{(cq/a, d/c; q)_\infty}{(cq, q/c; q)_\infty}\right\}. \qquad (2.2)$$

$${}_4\Psi_4\left[\begin{array}{l} a, b, cq, \lambda q; q; \frac{1}{a} \\ d, bq, c, \lambda \end{array}\right] = \frac{(b-c)(b-\lambda)}{b(1-\lambda)(1-c)}\,\frac{(q;q)_\infty^2(bq/a, d/b; q)_\infty}{(q/a, q/b, d, bq; q)_\infty}. \qquad (2.3)$$

Proof of (2.1)-(2.3)

Let us consider the Bailey's transform,

$${}_2\Psi_2\left[\begin{array}{l} a, b; q; z \\ d, c \end{array}\right] = \frac{(az, d/a, c/b, dq/abz; q)_\infty}{(z, d, q/b, cd/abz; q)_\infty}\ {}_2\Psi_2\left[\begin{array}{l} a, abz/d; q; d/a \\ az, c \end{array}\right]. \qquad (2.4)$$

[5; (5.20) (i), p.150]

Putting $c = bq$ and $z = q/a$ in (2.4) we have,

$${}_2\Psi_2\left[\begin{array}{l} a, b; q; q/a \\ d, bq \end{array}\right] = \frac{(q;q)_\infty^2(d/a, d/b; q)_\infty}{(q/a, q/b, d, d; q)_\infty}\ {}_2\Phi_1\left[\begin{array}{l} a, bq/d; q; d/a \\ bq \end{array}\right]. \qquad (2.5)$$

Summing the ${}_2\Phi_1$-series by making use of [5; App. II (II.8)] we have,

$${}_2\Psi_2\left[\begin{array}{l} a, b; q; q/a \\ d, bq \end{array}\right] = \frac{(q;q)_\infty^2(bq/a, d/b; q)_\infty}{(q/a, q/b, d, bq; q)_\infty}, \qquad (2.6)$$

which is a known result [3; (1.1) p. 165].

Now, let us consider

$${}_2\Psi_2\left[\begin{array}{l} a, b; q; 1/a \\ d, bq \end{array}\right] - b\ {}_2\Psi_2\left[\begin{array}{l} a, b; q; q/a \\ d, bq \end{array}\right] = \sum_{n=-\infty}^{\infty}\frac{(a;q)_n(1-b)}{(d;q)_n}\frac{1}{a^n}$$

$$= (1-b)\ {}_1\Psi_1\left[\begin{array}{c} a; q; 1/a \\ d \end{array}\right]. \tag{2.7}$$

If we make use of the summation formula [5; App. II (II.20)] to sum ${}_1\Psi_1$-series in (2.7) we get,

$$ {}_2\Psi_2\left[\begin{array}{c} a, b; q; 1/a \\ d, bq \end{array}\right] = b\ {}_2\Psi_2\left[\begin{array}{c} a, b; q; q/a \\ d, bq \end{array}\right]. \tag{2.8}$$

From (2.6) and (2.8) we find,

$$ {}_2\Psi_2\left[\begin{array}{c} a, b; q; 1/a \\ d, bq \end{array}\right] = b\frac{(q;q)^2_\infty (bq/a, d/b; q)_\infty}{(q/a, q/b, d, bq; q)_\infty}. \tag{2.9}$$

Now, consider

$$ {}_3\Psi_3\left[\begin{array}{c} a, b, c; q; 1/a \\ d, bq, cq \end{array}\right] - c\ {}_3\Psi_3\left[\begin{array}{c} a, b, c; q; q/a \\ d, bq, cq \end{array}\right] = (1-c)\ {}_2\Psi_2\left[\begin{array}{c} a, b; q; 1/a \\ d, bq \end{array}\right]. \tag{2.10}$$

From (2.9) and (2.10) we get,

$$ {}_3\Psi_3\left[\begin{array}{c} a, b, c; q; 1/a \\ d, bq, cq \end{array}\right] - c\ {}_3\Psi_3\left[\begin{array}{c} a, b, c; q; q/a \\ d, bq, cq \end{array}\right] = (1-c)b\frac{(q;q)^2_\infty (bq/a, d/b; q)_\infty}{(q/a, q/b, d, bq; q)_\infty}. \tag{2.11}$$

Again, interchanging b and c in (2.11) we get,

$$ {}_3\Psi_3\left[\begin{array}{c} a, b, c; q; 1/a \\ d, bq, cq \end{array}\right] - b\ {}_3\Psi_3\left[\begin{array}{c} a, b, c; q; q/a \\ d, bq, cq \end{array}\right] = (1-b)c\frac{(q;q)^2_\infty (cq/a, d/c; q)_\infty}{(q/a, q/c, d, cq; q)_\infty}. \tag{2.12}$$

Substracting (2.12) from (2.11) we get

$$\begin{aligned}(b-c)\ {}_3\Psi_3\left[\begin{array}{c} a, b, c; q; \frac{q}{a} \\ d, bq, cq \end{array}\right] &= \frac{(q;q)^2_\infty}{(q/a, d; q)_\infty} \\ &\times \left\{ b(1-c)\frac{(bq/a, d/b; q)_\infty}{(bq, q/b; q)_\infty} - c(1-b)\frac{(cq/a, d/c; q)_\infty}{(cq, q/c; q)_\infty} \right\} \end{aligned} \tag{2.13}$$

which is precisely (2.2).
Multiplying (2.11) by b and (2.12) by c and then Substracting second from first we

have,

$$(b-c)\ {}_3\Psi_3\left[\begin{matrix}a,b,c;q;\frac{1}{a}\\ d,bq,cq\end{matrix}\right]=\frac{(q;q)_\infty^2}{(q/a,d;q)_\infty}$$
$$\times\left\{b^2(1-c)\frac{(bq/a,d/b;q)_\infty}{(bq,q/b;q)_\infty}-c^2(1-b)\frac{(cq/a,d/c;q)_\infty}{(cq,q/c;q)_\infty}\right\} \qquad (2.14)$$

which is precisely (2.1).
Let us now consider,

$${}_3\Psi_3\left[\begin{matrix}a,b,cq;q;1/a\\ d,bq,c\end{matrix}\right]=\sum_{n=-\infty}^{\infty}\frac{(a,b;q)_n}{(d,bq;q)_n}\frac{1}{a^n}\left(\frac{1-cq^n}{1-c}\right)$$
$$=\frac{1}{1-c}\ {}_2\Psi_2\left[\begin{matrix}a,b;q;1/a\\ d,bq\end{matrix}\right]-\frac{c}{1-c}\ {}_2\Psi_2\left[\begin{matrix}a,b;q;q/a\\ d,bq\end{matrix}\right]. \qquad (2.15)$$

Using (2.6) and (2.9) in (2.15) we get,

$${}_3\Psi_3\left[\begin{matrix}a,b,cq;q;1/a\\ d,bq,c\end{matrix}\right]=\frac{(b-c)}{(1-c)}\frac{(q;q)_\infty^2(bq/a,d/b;q)_\infty}{(q/a,q/b,d,bq;q)_\infty}. \qquad (2.16)$$

which is a known result [4, page 305].
Now, let us consider

$${}_3\Psi_3\left[\begin{matrix}a,b,cq;q;1/a\\ d,bq,c\end{matrix}\right]-b\ {}_3\Psi_3\left[\begin{matrix}a,b,cq;q;q/a\\ d,bq,c\end{matrix}\right]$$
$$=(1-b)\ {}_2\Psi_2\left[\begin{matrix}a,cq;q;1/a\\ d,c\end{matrix}\right]$$
$$=\frac{1-b}{1-c}\ {}_1\Psi_1\left[\begin{matrix}a;q;1/a\\ d\end{matrix}\right]-\frac{c(1-b)}{1-c}\ {}_1\Psi_1\left[\begin{matrix}a;q;q/a\\ d\end{matrix}\right]. \qquad (2.17)$$

Summing ${}_1\Psi_1$ series in (2.17) by using [5; App. II (II.20)] we find,

$${}_3\Psi_3\left[\begin{matrix}a,b,cq;q;1/a\\ d,bq,c\end{matrix}\right]=b\ {}_3\Psi_3\left[\begin{matrix}a,b,cq;q;q/a\\ d,bq,c\end{matrix}\right]. \qquad (2.18)$$

Thus from (2.16) and (2.18) we have

$$ {}_3\Psi_3\left[\begin{matrix} a, b, cq; q; q/a \\ d, bq, c \end{matrix}\right] = \frac{(b-c)}{b(1-c)} \frac{(q;q)_\infty^2 (bq/a, d/b; q)_\infty}{(q/a, q/b, d, bq; q)_\infty}. \tag{2.19} $$

Again, proceeding by taking

$$ {}_4\Psi_4\left[\begin{matrix} a, b, cq, \lambda q; q; 1/a \\ d, bq, c, \lambda \end{matrix}\right] = \sum_{n=-\infty}^{\infty} \frac{(a, b, cq; q)_\infty}{(d, bq, c; q)_\infty} \left(\frac{1-\lambda q^n}{1-\lambda}\right) \frac{1}{a^n} $$

$$ = \frac{1}{1-\lambda} {}_3\Psi_3\left[\begin{matrix} a, b, cq; q; 1/a \\ d, bq, c \end{matrix}\right] - \frac{\lambda}{1-\lambda} {}_3\Psi_3\left[\begin{matrix} a, b, cq; q; q/a \\ d, bq, c \end{matrix}\right]. \tag{2.20} $$

Making use of (2.16) and (2.19) in (2.20) we have

$$ {}_4\Psi_4\left[\begin{matrix} a, b, cq, \lambda q; q; 1/a \\ d, bq, c, \lambda \end{matrix}\right] = \frac{(b-c)(b-\lambda)}{b(1-\lambda)(1-c)} \frac{(q;q)_\infty^2 (bq/a, d/b; q)_\infty}{(q/a, q/b, d, bq; q)_\infty}, \tag{2.21} $$

which is precisely (2.3).

3. Special Cases

In this section we deduce certain special cases of the results established in section 2.

(i) Taking $d = 0$ and $a \to \infty$ in (2.1) we get,

$$ (b-c) \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n-1)/2}}{(1-bq^n)(1-cq^n)} $$

$$ = (q;q)_\infty^2 \left\{ \frac{b^2}{(1-b)} \frac{1}{(bq, q/b; q)_\infty} - \frac{c^2}{(1-c)} \frac{1}{(cq, q/c; q)_\infty} \right\}. \tag{3.1} $$

Putting $b = e^{i\theta}$ and $c = e^{-i\theta}$ in (3.1) we get,

$$ \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n-1)/2}}{(1-2q^n\cos\theta + q^{2n})} = \frac{(2\cos\theta - 1)}{(1-\cos\theta)} \frac{(q;q)_\infty^2}{\prod_{n=1}^{\infty}(1-2q^n\cos\theta + q^{2n})}. \tag{3.2} $$

Taking $\theta = \pi/2$ in (3.2) we have

$$ \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n-1)/2}}{(1+q^{2n})} = -\frac{(q;q)_\infty^2}{\prod_{n=1}^{\infty}(1+q^{2n})} = -\frac{(q;q)_\infty^2}{(-q^2;q^2)_\infty^2}. \tag{3.3} $$

Taking $\theta = \pi$ in (3.2) we get,

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n-1)/2}}{(1+q^n)^2} = \frac{-3}{2} \frac{(q;q)_\infty^2}{\prod_{n=1}^{\infty}(1+q^n)^2} = \frac{-3}{2} \frac{(q;q)_\infty^2}{(-q;q)_\infty^2}. \tag{3.4}$$

Putting $\frac{\pi}{2} + \theta$ for θ in (3.1) we have

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n-1)/2}}{(1+2q^n \sin\theta + q^{2n})} = -\frac{1+2\sin\theta}{1+\sin\theta} \frac{(q;q)_\infty^2}{\prod_{n=1}^{\infty}(1+2q^n\sin\theta + q^{2n})}. \tag{3.5}$$

(ii) Taking $d = 0$ and $a \to \infty$ in (2.2) we get,

$$(b-c) \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n+1)/2}}{(1-bq^n)(1-cq^n)}$$
$$= (q;q)_\infty^2 \left\{ \frac{b}{(1-b)} \frac{1}{(bq, q/b; q)_\infty} - \frac{c}{(1-c)} \frac{1}{(cq, q/c; q)_\infty} \right\}. \tag{3.6}$$

(3.6) can be expressed as,

$$(b-c) \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n+1)/2}}{(1-bq^n)(1-cq^n)} = (q;q)_\infty^2 \left\{ \frac{(1-c)}{(c, 1/c; q)_\infty} - \frac{(1-b)}{(b, 1/b; q)_\infty} \right\}.$$
$$= (q;q)_\infty \left\{ \frac{(q;q)_\infty}{(1-1/c)(cq, q/c; q)_\infty} - \frac{(q;q)_\infty}{(1-1/b)(bq, q/b; q)_\infty} \right\}. \tag{3.6A}$$

Comparing (3.6A) with [2; (3.8) p. 34] we have,

$$(q;q)_\infty \left\{ \frac{b}{1-b} \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} N_v(m,n) b^m q^n - \frac{c}{1-c} \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} N_v(m,n) c^m q^n \right\}$$
$$= (q;q)_\infty \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} N_v(m,n) q^n \left\{ \frac{b^{m+1}}{1-b} - \frac{c^{m+1}}{1-c} \right\}, \tag{3.6B}$$

where $N_v(m,n)$ stands for the number vector partitions of n with crank m.

Putting $b = \lambda e^{i\theta}$ and $c = \lambda e^{-i\theta}$ in (3.6) we have,

$$2\lambda i \sin\theta \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n+1)/2}}{1 - 2\lambda q^n \cos\theta + \lambda^2 q^{2n}}$$

$$= (q;q)_\infty^2 \left\{ \frac{\lambda e^{i\theta}}{(1-\lambda e^{i\theta}) \prod_{n=1}^{\infty}(1 - 2\lambda q^n \cos\theta + \lambda^2 q^{2n})} - \frac{\lambda e^{-i\theta}}{(1-\lambda e^{-i\theta}) \prod_{n=1}^{\infty}(1 - 2\lambda q^n \cos\theta + \lambda^2 q^{2n})} \right\} \tag{3.7}$$

which on simplification gives

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n+1)/2}}{(1 - 2\lambda q^n \cos\theta + \lambda^2 q^{2n})} = \frac{(q;q)_\infty^2}{(1 - 2\lambda\cos\theta + \lambda^2)\prod_{n=1}^{\infty}(1 - 2\lambda q^n \cos\theta + \lambda^2 q^{2n})}. \tag{3.8}$$

Taking $\theta = 0$ in (3.8) we get

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n+1)/2}}{(1-\lambda q^n)^2} = \frac{(q;q)_\infty^2}{(1-\lambda)^2 \prod_{n=1}^{\infty}(1-\lambda q^n)^2}. \tag{3.9}$$

For $\theta = \pi/2$, (3.8) yields

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n+1)/2}}{(1+\lambda^2 q^{2n})^2} = \frac{(q;q)_\infty^2}{(1+\lambda)^2 \prod_{n=1}^{\infty}(1+\lambda^2 q^{2n})}. \tag{3.10}$$

For $\theta = \pi$, (3.8) yields

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n+1)/2}}{(1+\lambda q^n)^2} = \frac{(q;q)_\infty^2}{(1+\lambda)^2 \prod_{n=1}^{\infty}(1+\lambda q^n)^2}. \tag{3.11}$$

If we take $\lambda = 1$ in (3.11) we get,

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n+1)/2}}{(1+q^n)^2} = \frac{(q;q)_\infty^2}{2(-q;q)_\infty^2}. \tag{3.12}$$

Putting $\lambda = 1$ in (3.10) we get

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n+1)/2}}{(1+q^{2n})} = \frac{(q;q)_\infty^2}{2(-q^2;q^2)_\infty^2}. \tag{3.13}$$

(iii) Putting $d = 0$ and $a \to \infty$ in (2.3) we get

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n-1)/2}(1-cq^n)(1-\lambda q^n)}{(1-bq^n)} = \frac{(b-c)(b-\lambda)}{b(1-b)} \frac{(q;q)_\infty^2}{(bq, q/b; q)_\infty}. \tag{3.14}$$

Comparing (3.14) with [2; (3.8) p. 34] we have,

$$= \frac{(b-c)(b-\lambda)}{b(1-b)} (q;q)_\infty \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} N_v(m,n) b^m q^n,$$

where $N_v(m,n)$ is the number of vector partitions of n with crank m.
Taking q^5 for q and then putting $b = q^2$ in (3.14) we have

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{5n(n-1)/2}(1-cq^{5n})(1-\lambda q^{5n})}{(1-q^{5n+2})} = \frac{(q^2-c)(q^2-\lambda)}{q^2(1-q^2)} \frac{(q^5;q^5)_\infty^2}{(q^3,q^7;q^5)_\infty}$$
$$= \frac{(q^2-c)(q^2-\lambda)}{q^2} \frac{(q^5;q^5)_\infty^2}{(q^2,q^3;q^5)_\infty}. \tag{3.15}$$

Taking q^5 for q and then putting $b = q$ in (3.14) we get

$$\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{5n(n-1)/2}(1-cq^{5n})(1-\lambda q^{5n})}{(1-q^{5n+1})} = \frac{(q-c)(q-\lambda)}{q(1-q)} \frac{(q^5;q^5)_\infty^2}{(q^4,q^6;q^5)_\infty}$$
$$= \frac{(q-c)(q-\lambda)}{q} \frac{(q^5;q^5)_\infty^2}{(q,q^4;q^5)_\infty}. \tag{3.16}$$

From (3.15) and (3.16) and corollary [1; (6.2.6) p. 153] we have

$$\frac{\displaystyle\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{5n(n-1)/2}(1-cq^{5n})(1-\lambda q^{5n})}{(1-q^{5n+2})}}{\displaystyle\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{5n(n-1)/2}(1-cq^{5n})(1-\lambda q^{5n})}{(1-q^{5n+1})}} = \frac{(q^2-c)(q^2-\lambda)}{q(q-c)(q-\lambda)} \frac{(q,q^4;q^5)_\infty}{(q^2,q^3;q^5)_\infty}$$
$$= \frac{(q^2-c)(q^2-\lambda)}{q(q-c)(q-\lambda)} \left\{ \frac{1}{1+} \frac{q}{1+} \frac{q^2}{1+} \frac{q^3}{1+\ldots} \right\} \tag{3.17}$$

Taking $\lambda = c = 0$ in (3.17) we find,

$$\frac{\displaystyle\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{5n(n-1)/2}}{(1-q^{5n+2})}}{\displaystyle\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{5n(n-1)/2}}{(1-q^{5n+1})}} = \frac{q}{1+} \frac{q}{1+} \frac{q^2}{1+} \frac{q^3}{1+\ldots}. \tag{3.18}$$

(iv) Putting $\lambda = 0$ in (2.3) we get

$$ {}_3\Psi_3 \left[\begin{matrix} a, b, cq; q; 1/a \\ d, bq, c \end{matrix} \right] = \frac{(b-c)}{(1-c)} \frac{(q;q)_\infty^2 (bq/a, d/b; q)_\infty}{(q/a, q/b, d, bq; q)_\infty}. \tag{3.19} $$

Taking $d = c = 0$, $a \to \infty$ in (3.19) we get,

$$ \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n-1)/2}}{(1-bq^n)} = \frac{b}{1-b} \frac{(q;q)_\infty^2}{(bq, q/b; q)_\infty}. \tag{3.20} $$

Replacing q by q^2 and putting $b = q$ in (3.20) and using [1; (1.1.7), p. 11] we get,

$$ \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{n(n-1)}}{(1-q^{2n+1})} = q \frac{(q^2;q^2)_\infty^2}{(q;q^2)_\infty^2} = q\Psi^2(q). \tag{3.21} $$

Replacing q by q^8 and b by q in (3.20) we get,

$$ \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{4n(n-1)}}{(1-q^{8n+1})} = q \frac{(q^8;q^8)_\infty^2}{(q, q^7; q^8)_\infty}. \tag{3.22} $$

Again, replacing q by q^8 and b by q^3 in (3.20) we find,

$$ \sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{4n(n-1)}}{(1-q^{8n+3})} = q^3 \frac{(q^8;q^8)_\infty^2}{(q^3, q^5; q^8)_\infty}. \tag{3.23} $$

Taking the ratio of (3.22) and (3.23) and using [1; (6.2.38), p. 154] we get,

$$ \frac{\displaystyle\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{4n(n-1)}}{(1-q^{8n+3})}}{\displaystyle\sum_{n=-\infty}^{\infty} \frac{(-1)^n q^{4n(n-1)}}{(1-q^{8n+1})}} = \frac{q^2}{1+} \frac{q+q^2}{1+} \frac{q^4}{1+} \frac{q^3+q^6}{1+\ldots}. \tag{3.24} $$

(v) Taking $d = q$, $a \to \infty$ and $c = 0$ in (3.19) we get,

$$ \sum_{n=0}^{\infty} \frac{(-1)^n q^{n(n-1)/2}}{(q;q)_\infty (1-bq^n)} = \frac{b}{1-b} \frac{(q;q)_\infty}{(bq;q)_\infty} = \frac{b(q;q)_\infty}{(b;q)_\infty}. \tag{3.25} $$

Replacing q by q^3 and putting $b = q^2$ in (3.25) we get,

$$\sum_{n=0}^{\infty} \frac{(-1)^n q^{3n(n-1)/2}}{(q^3;q^3)_n(1-q^{3n+2})} = q^2 \frac{(q^3;q^3)_\infty}{(q^2;q^3)_\infty}. \tag{3.26}$$

Again, replacing q by q^3 and b by q in (3.25) we have,

$$\sum_{n=0}^{\infty} \frac{(-1)^n q^{3n(n-1)/2}}{(q^3;q^3)_n(1-q^{3n+1})} = q \frac{(q^3;q^3)_\infty}{(q;q^3)_\infty}. \tag{3.27}$$

Taking the ratio of (3.26) and (3.27) and using [1; (7.1.1), p. 179] we get,

$$\frac{\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n q^{3n(n-1)/2}}{(q^3;q^3)_n(1-q^{3n+1})}}{\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n q^{3n(n-1)/2}}{(q^3;q^3)_n(1-q^{3n+2})}} = q^{-1}\frac{(q^2;q^3)_\infty}{(q;q^3)_\infty} = \frac{q^{-1}}{1-}\,\frac{q}{1+q-}\,\frac{q^5}{1+q^2-}\,\frac{q^5}{1+q^3-\dots}. \tag{3.28}$$

Putting q^4 for q and $b = q$ in (3.25) we get,

$$\sum_{n=0}^{\infty} \frac{(-1)^n q^{2n(n-1)}}{(q^4;q^4)_n(1-q^{4n+1})} = \frac{q(q^4;q^4)_\infty}{(q;q^4)_\infty}. \tag{3.29}$$

Putting q^4 for q and $b = q^3$ in (3.25) we find,

$$\sum_{n=0}^{\infty} \frac{(-1)^n q^{2n(n-1)}}{(q^4;q^4)_n(1-q^{4n+3})} = \frac{q^3(q^4;q^4)_\infty}{(q^3;q^4)_\infty}. \tag{3.30}$$

Taking the ratio of (3.29) and (3.30) and using [1; (7.1.2) p. 179] we get

$$\frac{\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n q^{2n(n-1)}}{(q^4;q^4)_n(1-q^{4n+1})}}{\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n q^{2n(n-1)}}{(q^4;q^4)_n(1-q^{4n+3})}} = q^{-2}\frac{(q^3;q^4)_\infty}{(q;q^4)_\infty} = \frac{q^{-2}}{1-}\,\frac{q}{1+q^2-}\,\frac{q^3}{1+q^4-}\,\frac{q^5}{1+q^6-\dots}. \tag{3.31}$$

A number of similar results can also be deduced.

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2. Financial Inclusion and Demonetisation: Two Sides of the Same Coin Present Experience of Indian Economy

Dr. Karuna V. Shinde
Assistant professor, Department of Economics, Smt. Parmeshwaridevi Durgadutt Tibrewala, Lions Juhu, College of Arts, Commerce & Science.

Abstract

To become economic leader of the region and to achieving inclusive growth the government of India has taken up several measures for financial inclusion. However, it has been observed that these measures are not encouraging in its nature. And the first steps towards that movement India needs to eradicate black money, corruption and financial crimes. In this way Indian government adopted demonetization in November 2016 to tackle with black money and make India a cashless digital economy with the implementation of demonetization and considering the country's agenda to improve financial inclusion.

Along with this technological era and youth potential there is a scope to move for third phase of financial reforms. However, lack of financial inclusion and financial illiteracy are the major challenges before it.

Paper makes an attempt to study the effect of demonetization on financial inclusion and to find out how it influences the Indian Economy.

Keywords: Cashless Economy, Demonetization, Digital Economy, Financial Inclusion.

Introduction

Financial inclusion is an essential condition for achieving the uniform economic development with greater economic and social equality. But the presence of unaccounted money and parallel economy this extensive dream making it impossible .Therefore it is very necessary to bring the entire money within the purview of the law.

Financial inclusion is a process which ensure that all the financial products and services needed are adequately assess by all sections of the society in general and particularly weaker sections of the society at an affordable cost.

The main intention behind this is to make a banking services user friendly in terms of savings, credit and remittance needs particular and financial products and services general.

Accesses to credit and savings facilities offer the poorer financial security which make them capable to expand their business adequately manage consumption along with household expenses to deal with inflationary pressure.

Which help to improve the standard of living and poverty falls, allowing people to contribute more to the economy as well.

However it has been observed that the ignorance, lack of financial literacy and economic surveillance, India could not achieve 100% financial inclusion as planned in the past.

This is a warning for the economic supremacy in the region. Government is relentlessly pursuing the matter of black money. There was an urge for essential and severe action for complete inclusion. November 8, 2016 demonetization announcement is to be seen in this perspective.

Objectives

1. To analyse the role of financial inclusion in the growth of Indian banking system
2. To seek the role of demonetisation in promoting financial education
3. To understand how the demonetisation influencing the way of achieving full financial inclusion by 2015.
4. To access the Indian experience in the field of Financial Inclusion.

Review of Literature

To achieve continuous and sustainable growth of the nation, universal financial services are pre-requisite which play an important contribution in raising economic growth, reducing poverty and enhance economic opportunities which inculcate all the section of the society. (Dr. Joji Chandran, 2008).

Financial inclusion is a process which makes all the people to assess banking services at affordable cost which was earlier ignored the formal financial system. (Thorat, 2006)

Financial inclusion emphasising on lower strata of the society who are unable to enjoy all the banking services earlier which will help them to assess all the banking products easily and efficiently. (Leeladhar, 2006)

Indian scenario of Financial Inclusion

It has been observed that less than 50% of Indian population face the problem of poverty along with hunger out of which only 31% has able to access banking services. However 80% populations are not even covered under life and health insurance. By considering this fact it is realise that there is a wide scope of growth of Indian banking system, accordingly the India's national vision for 2020 has set up a goal to cover 600 million new customers in a banking services in form of Micro finance and Micro insurance. To make this vision successful following requisite are necessary:

1. Access Financial Market

The present scenario show that nearly 99 blocks in the country don't have any bank branches, in which 86 are in North east and 13 are in other parts of country. A sum of Rs150 crore sanction by the government for banks branch expansion in unbanked and difficult areas, obviously thin density of population in the north-east area it become very difficult for bank branches expansion. To solve this problem RBI has proposed to use branchless banking with help of technology to promote inclusion through micro finance bodies, business correspondent, co-operative societies, grocery shops etc. which will help to access easily the financial market.

2. Access Credit Market

To make needy people to access the credit market the ample avenue of financial products should be available. For that purpose, saving linked financial model can be adopted for these segment, which should have kept simple and guarantee the beneficiaries of credit limit. In the rural part of state, primary agriculture co-operative societies are preety active in many part of the rural areas of state. Thus, now state governments have taken initiating steps to rationalization functioning of co-operative societies in the area of procurement, fertilizers and pesticides sale etc.

3. Learn financial matters

Lack of financial awareness and poor infrastructure make the people financial excusive , in order to correct this situation RBI setting up the pilot project on 18th June 2007 based on the credit councelling and financial inclusion, accordingly multinational website introduced which help to provide knowledge to the farmers about banking fsacilities. To inculcate lower strata of the Indian society is the basic objective of the financial inclusion programme.

Initial Steps for Financial Inclusion

Before 1990 Reserve bank of India and government had taken several initiatives to promote the financial inclusion in India.

1. In the year 1955 there was a creation of State Bank of India
2. In the yer 1969 and 1980 Nationalisation of commercial banks.
3. Initiating the Lead Bank Scheme in 1970;
4. In the year 1982 National Bank for Agriculture and Rural Development (NABARD) was set up to provide refinance credit to agriculture,
5. To expand bank branches in the rural areas the Establishment of regional rural banks was established in the year 1975.
6. Regulation of the interest rate ceiling for credit in weaker sections.
7. The concept of the Self help groups linkage programmed was introduced by the by NABARD,in the year 1992 which facilitates and provides door step banking services to the financial exclused group of the vulnerable society.
8. Simplifications of Know your customer (KYC) norms are another milestone for the financial inclusion in which with the help of NGOs set up they organize the poor, make them capable to move in the process of empowerment.
9. To provide financial credit to the farmers in the year 1998 Kisan credit card has been launched.
10. As per the suggestion of NABARD in 2005 General credit card has been launched hich provide the facility to lend the money up to Rs. 25000/-.
11. NGOs, SHGs, and Micro Finance Institutions are permitted by RBI in the year 2006

Rationale of Demonetization

No doubt India is rapidly expanding increasing in terms of growth and hold the number one position in terms of growth however the in in Global Corruption Perception Ranking it hold the 76 place. The strong influence of the evil of corruption and black money influencing the system which make the efforts in financial inclusion weak.

It has been also observed that, existence of huge number of high value currency notes has created favourable environment for hoarding black money, corruption in business and politics

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It has been also observed that, existence of huge number of high value currency notes has created favourable environment for hoarding black money, corruption in business and politics and a source of funding of terrorism by unreceptive countries. Therefore demonetization was consider as the concrete solution for sterilisation of these evils.

Followings are the preliminary steps were taken during the last two years

- Link of Aadhar seeding to gas and other services
- Compulsory use of the PAN for high value transactions.
- Jan Dhan Yojana of no frills accounts.
- Income Exposé Scheme.

Effects of Demonetization

Overall impact of the demonetisation on the Indian economy show that there is sever disruption to the business due to which growth expectations have reduced. To 6.9% for the financial year 2017.

The following are the other effects on the economy:

- People who have earned money through illegal ways would be afraid to disclosed the money due the fear of prosecution as it is a illegitimacy of the income it automatically generate deflation in the market..
- It has lead the reduction of velocity of circulation money in the economy.
- A lot of legally earned cash eposited in the banks and excess supply of the bank money enable the banks capable to make more lending which altimately cut down the rate lending
- It become easy to access the loans and it help to boost the economic activities as the interest rates reduces

Advantages of Demonetization

- Control over practise of black money and corruption to large extent
- Sterilised activities of arms smuggling and terrorist activities.
- Imposing the withdrawal limits set by the RBI reduced currency circulation. Which developed the practise of cashless transaction.
- It help the government to track the money as the regulations for exchange of money in banks emphasising on showing valid identity card like PAN, Aadhar card and electoral card.
- Financial Intelligence Unit keep track all details of the transactions which creat difficult task for hiding black money.

- Real estate industry attract the foreign investors as well as domestic investor, due to the practise of more transparency and credibility.

Disadvantages of Demonetization

- It make more inconvenience to common man for exchanging old high denomination notes.
- Replacing all the old high denomination notes, create costly affaire for the RBI.
- The general business activity has saturated which resulting thousands of crores of loss to the national income.
- Create adverse impact of the half of the country population who are not well aware about credit transaction.
- The major problem is that big fishes will be left out whose who invested black money in the form of gold and property and foreign currency, and hidden in tax havens are not trap by the regulation body of the RBI
- The ATM recalibration will become time consuming.

Demonetization as A tool of Financial Inclusion

Demonetization help to attain the fast practise of financial inclusion. It developed the rapid banking education to the vast multitude who was earlier uncovered due to financial exclusion. No doubt demonetization has ceased all regular banking business operations and loss of revenues temporarily, but for the future it served the aim of financial inclusion.

Conclusion

The fruit of demonetization are much encouraging and it is in the long term interest of the country. However it is very necessary that Government need to ensure that there will be a smooth flow of currency exchanges.

As it will have a massive impact on parallel economy. Though we feel it is a pain full process for the general masses of the country but this pain it is temporary, it taught financial lessons. It will help to control corruption, elections and terrorism for the long term in a country which is the main intension behind it. It is a reaping time for the banks that made considerable investments on digitization of banking services. The cashless and transparent mechanism has gained force post demonetization, which has led to increased financial inclusion and this force should be continued till India achieves complete financial inclusion.

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7. Global Education : A Challenge for the Indian Education System

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Assistant Professor, Department of Economics, Smt. Parmeshwaridevi Durgadutt Tibrewala Lions Juhu College of Arts, Commerce & Science.

Introduction

Globalisation is consider as an irresistible and benign force for transferring economic prosperity to people throughout the world, at the same time many thinkers believe that it is a source of all contemporary ills.

We can say that it is a process of transfer, adoption and development of values, knowledge, Technology and behavioural norms across countries and societies in different parts of the world. In simple term we can elaborate it as a global networking, which has strong influence on political, social and economic aspect of the nation.

It has been observed in practise that today's education system not able to face the current social challenges, as the youth of the country unable to cope up with the working world challenges, drug abuse, expansion of the poverty, Juvenile delinquency and violence etc.

Therefore it is very necessary to integrate into the world economy by developing the capabilities of acquiring new skills which is demanded by the knowledge society along with the foundation of the traditional knowledge and its tools.

Education is at the heart of the reforms in the field of science, culture, economics technology, and if such education associated with the path of globalisation it will open the new horizon of skills knowledge to the people of the country, which will empower them to access the new opportunities of the global world.

The dream of a prosperous global career attract many Indian students to enrol for the international education from reputed institutions in the world. Acquiring practical skills along with the knowledge is the pre requisite for the global workforce. Obviously Immense effort, determination required for working at such a level.

Globalisation give the new vision of lifelong learning in the following ways.

- Learning to know (Acquiring new skill)

- Learning to do (Accepting the new challenges)
- Learning to live together (Interdependence)
- Learning to be (Grooming personality by accepting global culture)

Accepting and adoption of the globalisation for any country is not an easy task as many views are connected with it. On one side it has been observed that many countries has taken efforts to adopt the globalisation with the intension at taking opportunities to develop their societies and people, on the other hand various social movements have been initiated against the treats of globalisation, especially the developing countries, therefore it is very important to examine the positive and negative side of the globalisation.

Positive impact of the globalisation

- Promote multiple developments at different levels due to global sharing of knowledge, skills, and intellectual capital;
- Solidarity, supplement and benefit to produce collaboration for various developments of countries and its societies.
- It help to create and enhance the values and efficiency to serving local needs and growth;
- Encouraging global understanding, collaboration, harmony, and acceptance to cultural diversity through countries and regions.
- Facilitating communications, interactions, and encouraging multi-cultural contributions between the various nations.

Negative impact of the globalisation

- It leads the technological and digital disparities between advanced countries and less developed countries;
- It leads appropriate opportunities for a few advanced countries as compare to the rest of the world.
- Expand inequalities and conflicts between areas and cultures;
- Promoting the dominant of developed countries in terms of cultures and values.

However many social thinkers believe that education is one of the fundamental vocal factor which has capacity to divert this negative impact into the positive impact through the conversion of the threat into opportunities for the development of the individual and societies.

But the important is that how to maximise the positive effect and minimise the negative impact of it, which help to minimise the global inequalities.

Education system of the every nation influencing by their own cultures, frame of the understandings and various other impactful scenarios that differ from a nation to another, however their sole purpose is somewhat similar. But still it has been observed that the standard of the education system is different.

Difference between Indian education system and Global education system

The education system in India is very different from the education system in the other countries. Rigid education system in India will restrict the student ability to develop the practical learning knowledge, as the roots of the Indian education system originate from the British education system itself, it resulted inadequate requisite physical and financial infrastructure.

Therefore many student prefer to go for the global education as the global education inculcate following aspects

- A practical and research-based approach to education in abroad destinations while Indian education system based on the theory
- Global educational organisation provides Funds facilities and investment in research education while Indian education provided either fully by the public authority or jointly by the public and private authority or solely private authority.
- A rich and diverse curriculum create an opportunities for the students to come out of their comfort-zone and limited knowledge zone and encourages them to do new and innovative research while Indian education system have a very rigid framework which is comple
- Flexibility on the choice of subjects
- A blend of conventional and contemporary programs like game designing, photography, mechatronics etc.

Foreign education system offers a personify, unconventional approach towards studies develop educational and intellectual skills and knowledge base of the subjects which they prefer to chosen.. It will enable them to become competitive for the global opportunities.

On the other hand the Indian rigid Education system engaged the students in the path of myriad of examinations and such conditions make them unable to enhance the skill essential for the global competition.

No doubt many concrete steps are being taken by the government to reform the Indian education system but yet there is a long way to reache the path of standard education, which will provide an opportunities for the Indian youth to get an experience connect with the real world.

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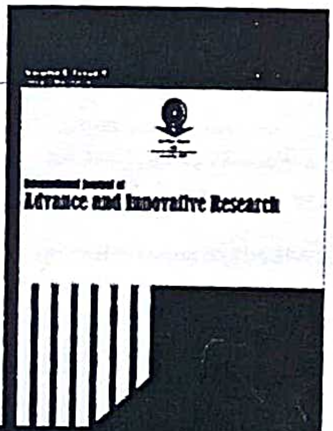

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6th International Multidisciplinary Conference (Online)

On

Innovations in Business, Financial Services and Education using Digital Technology

On

11th February, 2023 Saturday

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Organized by

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Schedule of Online Conference (Using Zoom Platform)

Timing	Particulars
09.15 am	Conference Registrations and Tea
09.45 am	Welcome Address by Conference Co-Convener Dr. Vanshika Ahuja & Asst.Prof Sona Dawra
09.50 am	Virtual Lamp Lighting
09.55 am	Introduction of Conference by Convener & Principal Dr. Gulabchand Gupta
10.05 am	Address by Guest of Honor Dr. Amit Kauts, Dean, Faculty of Education, GND University, Amritsar
10.25 am	Address by Guest of Honor Dr. Anup Lohani, Ex Research Scientist Robert Bosch, Research and Technology Center, Asia Pacific, Singapore
10.35 am	Address by Guest of Honor Dr. Vivek Patil, Principal Royal College, Dombivli, Dist Thane
10.45 am	Address by Guest of Honor Dr. Ameya Tripathi, Professor of Computer Engineering and Dean Research & Development, Don Bosco Institute of Technology, Mumbai.
11.00 am	Address by Chief Guest & Keynote Speaker Dr. R K Jain, Professor and Dean Research Studies, Oriental University Indore
11:20 am	Release of Conference Abstract Book
11.25 am to 01.00 pm	Paper Presentation Track 1: Chair by Dr. Ameya Tripathi and Dr. Sapna Modi Paper No from 1 to 20
01.00 pm to 02.00 pm	Lunch Break
02.00 pm to 03.45 pm	Paper Presentation Track 2: Chair by Dr. Sharmila Rathod and Dr. Natika Poddar Paper No from 21 to 35
03.45 pm to 04.00 pm	Tea Break
04.00 pm to 05.00 pm	Paper Presentation Track 3: Chair by Dr. Sharmila Rathod and Dr. Natika Poddar Paper No from 36 onwards
05.00 pm to 05.15 pm	Valedictory & Soft Copy Certificate Distribution
05.15 pm to 05.30 pm	Vote of Thanks

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Research Papers

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USE OF TECHNOLOGY IN ACADEMIC COMMUNICATION

Dr. Nidhi Pundir
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ABSTRACT
In this age of globalization, use of technology is becoming extremely important for everyone. During Covid -19 we shifted our teaching & learning, on online mode definitely some of us were not comfortable with the technology but we also got an opportunity to know about new technology such as digital media, new apps , new web tools .With the help of these web tools our teaching & learning became more interesting and easy. Through this research paper, I tried to focus on the concept of technology in academics how it is beneficial for us I also tried to give references of some educational web tools along with the limitations of technology for academic communication.

Keywords: technology, academicians, web tools, teaching & learning etc.

INTRODUCTION
The word technology comes from two Greek words, transliterated techne and logos. Techne is the Greek word for art, skill, craft, as well as the method or process by which something is acquired. The Greek term logos translates as "word," "utterance," "statement," or "expression." Therefore, technology literally refers to statements or conversations concerning how things are acquired. David Warlick who was an educator, author and early adopter of technology in education, He said," We need technology in every classroom and in every student and teacher's hand, because it is the pen and paper of our time, and it is the lens through which we experience much of our world. "

He emphasised that We as teachers must prepare our students for future. Indian Government, through NEP (National Education Policy) is also trying to highlights the importance of technology in academics. A much-needed revolution in Higher education system through the New Education Policy — to prepare the students for the rapidly evolving digital landscape in the global ecosystem. Various scientific and technological advances have made it imperative for colleges to reinvent learning methods and techniques.

The new education policy asserts on an experiential, integrated, student-focused, collaboration-based and analysis-driven pedagogy, aiming at holistic development of the student. Technology will play a huge role in achieving these objectives. To keep up with the digital transformation across the globe, educators must use technology as the key instrument to shape the future of our students. New-age technologies such as artificial intelligence is believed to be the future of learning.

OBJECTIVES OF MY RESEARCH PAPER ARE
- To analyse the impact of technology for effective academic communication. (There are great benefits that could be achieved by implementing technology.)
- To identify appropriate uses of various technologies tools to improve academic communication. (What are these web tools and how they are useful for us.)
- To discuss advantages as well as limitations of technology for communication.

RESEARCH METHODOLOGY
It is descriptive research based on secondary data collected from various sources like books, journals, magazines, government websites, articles in newspapers etc.

Impact of Technology for Effective Academic Communication
Technology provides academicians with easy-to-access information, Now a days everything is on our fingertips if you want to search anything within seconds with the help of information providers apps, or search engines anytime, anywhere. we can access information. It also gives students opportunities to practice and learn with fun. It enables students to explore new subjects and deepen their understanding of difficult concepts. The most important element that supports the use of technology in the educational system is the Internet.

1. Empowering Educators
Technology also gives educators free space from their routine tasks that add value to the quality of teaching. Any tool that saves time and effort of teachers leads the education quality better. Teachers can engage students meaningfully; they can organise learning materials which students can access independently and easily.

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Educators can prepare online resources and educational websites -to enhance virtual learning experience. Due to covid we all teachers and students were involved in online work and learnt about new tools and technology which makes our teaching & learning more interesting and meaningful.

2. Continuous Tracking of Learning Outcomes

Educators can track individual progress effectively. Technology enables educators to shift towards competency-based assessments. Online evaluation tools, online quizzes are excellent examples of it.

3. Experiential and Discovery-Based Teaching

Integration of technology helps to link theory with practice and develop valuable, lifelong skills and strategies. Online technologies allow students to collaborate and share ideas and discoveries. Educators must emphasise conceptual understanding rather than rote learning. Teachers can use 3D videos and models to explain difficult concepts, and help students build stronger fundamentals.

4. Integrating Vocational Education

Educators can use simulations and make students work on vocational projects replicating real-life situations in the virtual world. Students can also enrol in online internships and online courses in various fields, such as, architecture, engineering. There are various platforms for students as well as teachers where they can enrol themselves and at their comfortable place & time zone they can learn and update their knowledge such as Swayam, NPTEL, MOOC, Coursera are few examples of online learning platform.

Now a days A to z of new generation academicians is like this

A- Academia.edu./ Abode Acrobat Reader, **B-** Blog / Browser, **C-** Class Dogo, **D-** Digital Videos, **E-** Electronic -Mail , **F-** Facebook , **G-** Google Assistance tools , **H-** Hubspot . **I-**Internet, **J-** Java, **K-** Kindle, **L-** Linkedin, **M-** Microsoft. **N-** NVD3, **O-**Online Evaluation tools, **P-**PDF (Portable document format), **Q-** Qucickheal, **R-** Research Gate, **S-** Slide share, Skype, Swayam, **T-** Teacher tube, Twitter, **U** – URL, **V-** Vimeo based tools, **W-** Whats App, **X-** X-player, **Y-**You Tube, **Z-** Zoom, Zip

Few Useful Tools and Their Use for Academicians

- **Slide Share:** Slide share is an online tool which provides us educational content - PDFs, PowerPoint slides, videos, and others - as a presentation. displays can then be searched, viewed and shared by using each person. Slide share is the most famous presentation sharing website among academicians.
- **Ted Ed (Podcast):** Podcast is very useful to improve listening skills. We can listen world's greatest educators, researchers, and community leaders. These share their stories and motivate the listeners. It is a platform where easily we can get inspirational speeches from different areas.
- **Blogs:** Academicians may create their own blogs using lessons, links, pics of activities, travel blogs. Teachers can also promote assignment skills by giving assignments to students based on teacher's post.
- **You tube:** From every stream, for every subject academician can easily access lectures of experts on you tube.
- **Class Dojo:** It is useful for communication between teachers and parents. Teachers can upload photos and videos. It is generally used to share report of students to their parents.
- **Facebook:** It is a social media app which is also helpful for academicians, as teachers can create a separate page or account for teacher and students for educational purpose.
- **Google forms:** Google forms is generally used to create surveys, feedback, questionnaire for research, online quizzes. there may be a variety of question sorts you can use to make your very own quizzes.
- **Research Gate:** ResearchGate is social networking site for scientists and researchers to share papers, and locate collaborators.

Technology is all around us however, once the academicians understand how these tools work, they will have the ability to make efficient use of them in their teaching & learning process. We, as academicians can achieve success in our academic goals by climbing the ladder of these web tools. These web tools present a holistic way of teaching and learning.

LIMITATIONS OF TECHNOLOGY FOR COMMUNICATION

Technology is considered to be the best and at the same time the worst invention of all. However, it has caused many limitations.

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- As Lack of resources and digital literacy is a major issue to integrate media in classrooms in India. Specially in rural areas it's difficult to get resources such as internet, computers, projectors etc.
- Training Is required to the teachers to properly plan their class by integrating Media. Without proper training or workshops, it is difficult to deal with these educational tools so higher authority should organise training programs for teachers.
- Specially in case of social media students can waste their lots of time If teacher fails to help learner to decide their learning goals. Students can use media for non-academic purposes which leads to students' waste of time.
- Due to online learning most of the students are not comfortable with carrying books and writing notes.so it is becoming a risk to the traditional book and handwriting methods.
- Managing courses online is difficult. Some time it's difficult to access video or e-content.
- Not accessible everywhere. Due to network problem or without electricity it's not possible to access information.
- Implementing computers and the internet is expensive. Without financial assistance it's difficult to have computers & internet connection.
- Sometime we get misleading and misguiding information. We get wrong information if we are not accessing from reliable source.

CONCLUSION

To conclude my paper with the words of **Mahatma Gandhi**

"Live as if you were to die tomorrow,

Learn as if you were to live forever"

Desire to acquire knowledge is must for the academicians. We, as teachers should make every effort to remain update in our subject area. Be a Tech-Persona, "if you are not willing to learn, no one can help you, if you are determined to learn, no one can stop you. As earlier I mentioned Undoubtedly, there are daunting challenges to overcome before we can truly implement NEP across the nation. The dream of an equitable education system, ensuring highest-quality education to all, can be realised by making technology 'the ultimate partner'.

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16. Effectiveness of Job Shadowing Training for Internship in India Special Reference to Mumbai

Dr. Nanda Indulkar
Assistant Professor in Commerce, Smt. Parmeshwaridevi Durgadatt Tibrewala L. J. College of Arts Commerce & Science, J. B. Nagar Andheri (East), Mumbai.

Abstract

Internships are offered to the students to learn the practicality of the curriculum they are studying. It is meant to bridge the gap between what they have learn through books and how to implement the same practically. Job shadowing helps interns to observe work of the experts in their own field. This makes them aware of the process of work first then slowly grasp the idea of it and learn it. Job shadowing method makes them realize what they should do and what they should not do to complete the work effectively. Total 40 students gave their valuable responses to the questionnaire related to the effectiveness of job shadowing training for internship. Maximum respondents find it effective that this method at internship training helps them to understand which career path they want to choose.

Keywords: Job Shadowing, Internship, Training technique, On the Job Training, Host and Shadow

Introduction

Indian government has been working hard to achieve the successful implementation of the skill development programs for the better job opportunities[i]. They have been creating awareness among youth to find their passion and pursue what they are good at. This gave rise to sudden demand in the courses which are more of skill based, choice based to improve overall position of student.[ii] Students who are confused in their early days of college life about what to do next as there is huge gap between what they are learning and what they will be doing at the work. It has been seen that most of the students are taking admissions in particular course either under influence of their parents or friends[iii]. Only after completion of course they realize that this course will not benefit them much. In such situation, they either try to accept the reality or find other courses which can offer more weightage to their resume.

Many companies are offering internship programs where students can work in their vacation time and learn about the practicality of the field they want to pursue[iv]. It is also crucial time where they can decide what they want to do in the future. This helps students to learn the insights of the work and also understand their own capabilities while completing the work[v]. When students understand their own pros and cons , they can decide best for their own career path. They have better knowledge of the career development options available for them. It helps them choose what is best for them.

Job shadowing method helps them to understand practical work by observing before doing it. Students, who are aware of importance of management, can effectively use observation method to learn new things. It is the easiest and most economical way of training new employees to perform job is the "assigned apprentice" approach.[vi] It is also said that this approach has limitation as there is one person teaching how to do things from his perspective. But it is still effective, innovative way of teaching new things to the students who are completely unaware of the practical work.

Meaning and Definition

"Job shadowing is a type of on-the-job training that allows an interested employee to follow and closely observe another employee performing the role. This type of learning is usually used to onboard new employees into an organization or into a new role. Job shadowing may also be used as a learning opportunity for interns or students to gain an understanding of the role requirements and the job tasks."[vii] Job shadowing is methods where people can observe the work of someone else and try to understand where they can find interest in the same field. There are two parties involved in this method i.e. host and shadow. Host is the person who is expert in particular work and shadow is the student who wants to learn the skill from host. It is important to learn in advance what is expected from them and what kind of responsibilities they should fulfill at work place. This will help students to find out if they are meant for this kind of job or not.

"Internship is a period of time during which students work for a company in order to get experience of particular type of work."[viii] It is short period of time student spend in the organisation to gain knowledge and experience in particular field. Such internships can be paid or unpaid depends on nature of job.

Benefits of Job Shadowing

Job shadowing method is used for limited time duration at workplace. It helps people come together and share their views about work and help to continuous improvement. It makes people realize whether their current skills match with their career interests. It facilitates to find the competent person in same field whom you can follow and gain valuable information.

Job shadowing helps host to share work with his shadow. This can also give opportunity to the shadow to present his skills in front of host by doing work practicality.[ix] It helps to create network at work place and smoothens the barriers at work. It also facilitates people to look for better opportunities and self-development.

Objective of the Research

- To find out if students who have completed their internship at company have used job shadowing method or not
- To understand if students think that job shadowing method is effective for choosing right career or not.

Hypothesis

- H1 Job Shadowing is effective method for Interns to understand their career choice
- H0 Job Shadowing is not effective method to understand their career choice

Research Methodology

Primary data collection has been collected with the help of questionnaire method from total students who have completed internship in the company studying commerce and management at the college situated in Mumbai. Random sampling method is implemented for collecting the data.

Secondary data collection has been collected through books, essay, website, articles, blogs and research papers etc.

Finding

There are total 40 respondents who have given their opinion on the job shadowing training method and its implementation for the internship programs. 97.5% of respondents said that companies where they completed their internship are implementing on the job training method for the better understanding of the job and only one respondent said that there is off the job training method implemented in the company he worked.

92.5% of the respondents are aware of the job shadowing method as it has been implemented in the company for the training and only 7.5% of the respondents are not aware of this method.

40% of the respondents are strongly agree and 40% of the respondents agree to the effectiveness of the job shadowing training methods. 17.5% of the respondents are neutral about the effectiveness of method and only 2.5% of respondents do not agree to the effectiveness of the job shadowing method.

Do you agree that job shadowing method will help interns to choose right career?
40 responses

35% of the respondents strongly agree and 45% of the respondents agree that job shadowing training method help interns to understand their career choice in better way. 17.5% of the respondents are neutral about the job shadowing method can help to make career choice. and 2.5% of the respondents do not agree that job shadowing method will help to chose right career.

25% of the respondents strongly agree and 60% of the respondents agree that job shadowing training method helps employees in their career development. 10% of the respondents are neutral about jobs shadowing can help them in career development and 5% of the respondents do not agree that job shadowing method will help them in their career development.

Do you agree that companies who implement this job shadowing method will have less wastages?
40 responses

27.5% of the respondents strongly agree and 55% of the respondents agree that job shadowing training method is effective for organization to reduce wastages. 10% of the respondents are neutral about the job shadowing method can reduce wastages and 7.5% of the respondents do not agree that companies who implement job shadowing method will have less wastages.

Conclusion

From the above finding, it can be seen that maximum respondents have used job shadowing technique for internship. They find it effective for training new students who are want to be familiar with the work. . Observation is the key for success of the Job shadowing, if training is not provided properly then it will not be effective. In this method, students has to observe their seniors while doing job which helps to reduce cost as interns are not allotted to the task immediately. Job shadowing training method helps organizations to control wastage. Interns are given work related to their area of interest which makes them understand practical side of it. This method is easy to implement, economical and create better relation between host and shadow. This method makes people take decision for their work life, in which field they want to grow and if they really feel passionate about it.

Suggestions

- Virtual job shadowing is the latest technique used by students to find out about what field they should choose in future.
- Parents should be made more aware about the job shadowing which can help students to focus about what they want to pursue than randomly choosing something which was influence by others.
- Job shadowing is mostly done with the help of connection development in the particular field. Students should be directed positively to learn about this.
- It is important to find out the expert in the particular work and learn from observing them as how they complete their work effectively.
- Companies should train employees to observe experts more than telling employees what to do verbally.

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